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Jordan Gans-Morse, Sebastian Mazzuca, and Simeon Nichter

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Abstract

During elections in many countries, political parties distribute particularistic benefits to individuals. The existing literature reveals that parties choose from at least five distinct strategies when distributing benefits, but fails to explain how parties allocate resources across these strategies. Our formal model provides insight into this key question. Most studies focus exclusively on “vote buying,” a strategy by which parties reward voters for switching their votes. Our model first shows how parties trade off between “vote buying” and “turnout buying,” a strategy by which parties reward supporters for showing up at the polls (Nichter 2008). We then show how parties combine these and other commonly observed strategies.

1 Introduction

During elections in many countries, parties distribute particularistic benefits to individuals. Political operatives frequently hand out not just cash, but also a wide range of goods and services such as bags of rice, chickens, whisky, clothing, soccer balls, Viagra, haircuts, and teeth cleaning (Schaffer 2007: 2). This practice is typically called "vote buying," but it actually encompasses numerous distinct strategies. For example, parties can reward individuals for switching their votes, showing up at the polls, or even staying at home on Election Day. How do parties choose amongst these and other strategies when offering rewards during campaigns?

This question is particularly salient because it is part of a broader literature that extends well beyond electoral campaigns. Scholars continue to vigorously debate about how parties distribute targetable goods, such as infrastructure projects and particularistic benefits. Two seminal formal studies offer conflicting predictions: whereas Cox and McCubbins (1986) argue that parties will distribute targetable goods to core supporters, Lindbeck and Weibull (1987) contend they will target swing voters. A more recent paper by Gary Cox (2006) argues that these and other studies focus too narrowly on *persuasion* (changing voters' preferences); when strategies such as *mobilization* (affecting whether citizens vote) are considered, the core-supporter hypothesis is substantially strengthened. Building on these studies, we develop a model that explicitly incorporates both persuasion and mobilization strategies.

The existing literature highlights at least five strategies that parties employ when distributing particularistic benefits during elections. These strategies are shown in Figure 1, which adapts the typology in Nichter (2008). Each strategy targets individuals with different party preferences and likelihood of voting. We present the five strategies in the order that they are discussed in our paper:

Figure 1: Strategies for Distributing Targetable Goods

Recipient Inclined to Vote or Not Vote		Political Preference of Recipient vis-à-vis Party Offering Goods: *Favors Party*	Political Preference of Recipient vis-à-vis Party Offering Goods: *Indifferent or Favors Opposition*
	Inclined to Vote	**"Rewarding Loyalists"**	**"Vote Buying"** **"Negative Turnout Buying"**
	Inclined Not To Vote	**"Turnout Buying"**	**"Double Persuasion"**

Source: Adapted from Nichter (2008).

1. **Vote Buying:** Rewards opposing or indifferent voters for switching their votes. This strategy can increase votes for the machine and decrease votes for the opposition.

2. **Turnout Buying:** Rewards supporting nonvoters for showing up at the polls. This strategy can increase votes for the machine.

3. **Double Persuasion:** Rewards opposing or indifferent nonvoters for turning out and voting for the party. This strategy can increase votes for the machine.
4. **Negative Turnout Buying:** Rewards opposing or indifferent voters for not voting. This strategy can decrease votes for the opposition.

5. **Rewarding Loyalists:** Offers benefits to supporters who vote for the party even without rewards. This strategy is not incorporated in our model, and its impact is uncertain.

Before developing a formal model, we first briefly discuss existing studies on each of these five strategies. We also highlight the monitoring requirements of each strategy.

2 Reward Strategies

2.1 Vote Buying

The vast majority of studies on this topic focus on "vote buying." In this strategy, parties reward opposing or indifferent voters for switching their vote choices. Vote buying typically requires parties to have at least some ability to monitor specific vote choices. Otherwise, opposing voters could simply accept rewards and then vote for their preferred candidates.

Vote buying has existed across the world for many years. The practice was common in ancient Rome, where it was called *ambitus* (cf Schaffer 2007: 3). Before the introduction of the secret ballot in the U.S., newspapers would publish the cost of buying votes "just as they printed market reports on the prices of hogs and corn" (Cox and Kousser 1981: 654). In contexts with ballot secrecy, parties often develop clever ways to monitor vote-buying agreements. Parties in the Philippines give out carbon paper so voters can copy their ballots, whereas Italian parties lend mobile phones with cameras so reward recipients can photograph how they vote (Schaffer and Schedler 2007: 30-31).

Many studies provide empirical evidence of vote buying, but typically fail to distinguish whether parties are actually engaging in vote buying or one of the other four strategies discussed below. Stokes (2005: 315) offers a formal model and empirical tests to suggest that the Argentine Peronist party engages in vote buying, using its "deep insertion in voters' social networks" to monitor voters. Similarly, Cornelius (2004) provides strong evidence of vote buying in Mexico, and finds that lower income individuals in urban areas are most likely to be targeted.

2.2 Turnout Buying

Parties may also engage in "turnout buying" (Nichter 2008). In this strategy, parties distribute rewards to unmobilized supporters in exchange for showing up at the polls (Cox 2006; Nichter 2008; Hawkins and Rosas 2008; Dunning and Stokes 2008). Unlike vote buying, the strategy does not require monitoring of specific vote choices. Instead, turnout buying requires monitoring *whether* rewarded individuals vote.

Nichter (2008) provides a formal model of turnout buying and evidence from the United States and Argentina. During the 2004 US election, operatives in East St. Louis offered cigarettes, beer, medicine and money to increase turnout of the poor. One party official pleaded guilty and testified that operatives offered rewards "because if you didn't give them anything, then they wouldn't come out" (cf Nichter 2008: 19). In the case of Argentina, empirical tests suggest that survey data in Stokes (2005) are more consistent with turnout buying than vote buying (Nichter 2008).

Empirical studies of turnout buying are relatively new. In Venezuela, Rosas and Hawkins (2008: 1) find that targetable private goods are used to "secure victories by turning out loyal voters." In addition, Dunning and Stokes (2008) provide evidence from Argentina and Mexico that parties engage in both turnout buying and vote buying.

2.3 Double Persuasion

Another strategy is "double persuasion," which targets indifferent or opposing non-voters. This strategy provides rewards to both influence vote choice *and* induce participation. Double persuasion requires monitoring of both turnout and voting decisions.

As Chubb (1982: 171) explains in her study of clientelism in Italy, "many among the urban poor remain so totally alienated from the political system that they see no particular reason to prefer one party or candidate over another." The broader literature on clientelism suggests many individuals have little in the way of ideological preferences or reasons to vote, outside of the material reward structures set up by parties and candidates. During campaigns, parties can employ double persuasion to obtain these individuals' votes. Unlike the *swing voters* often targeted with vote buying, indifferent nonvoters will not show up at the polls without incentives. And unlike the *unmobilized supporters* targeted with turnout buying, they do not inherently prefer the machine on ideological grounds.

Nichter (2008) points out that studies of electoral rewards tend to ignore double persuasion, and highlights the need for more research focused on this important strategy. A recent paper by Dunning and Stokes (2008) actually suggests that double persuasion is a "perverse strategy." By contrast, we develop a model below that predicts that parties should be expected to engage in this strategy.

2.4 Negative Turnout Buying

Parties may also engage in "negative turnout buying," which rewards indifferent or opposing individuals for not voting (Cox and Kousser 1981; Heckelman 1998; Kornblith 2002; Morgan and Vardy 2008; Schedler 2002).[1] Similar to turnout buying, this strategy only requires monitoring whether or not rewarded individuals go to the polls, not actual vote choices.

An influential article by Cox and Kousser (1981) finds a marked increase in negative turnout buying, or what they term "deflationary fraud," after the introduction of the secret ballot in the United States. The authors examine references to rural election fraud in newspapers across New York State between 1879 and 1908. Cox and Kousser (1983: 662) conclude that "once delivery on the sale of a ballot became nearly impossible to verify, market transactions shifted...many more people were apparently paid to stay home after than before 1890."

Scholars also provide numerous examples of negative turnout buying in developing countries. For example, in both Guyana and Venezuela, operatives distributed

[1] This strategy is often called "negative vote buying" in the extant literature. However, building on Nichter (2008), we believe the term "negative turnout buying" is more precise as the strategy aims to influence turnout instead of vote choices.

rewards to opposition supporters in exchange for their voter identification cards (Schedler 2002: 4; Kornblith 2002: 9-11). In Mexico, Cornelius (2004: 53) reports that "arguably the most serious kind of abuse in the 2000 federal election was the purchase or 'renting' of voter credentials." In the Philippines, campaign workers rewarded potential opposition voters for dipping their fingers in ink (thus disqualifying them from voting) or taking bus trips out of town (Schedler 2002: 78). Such actions inhibit potential voters from exercising their right to vote, and can thereby reduce opposition turnout.

2.5 Rewarding Loyalists

By "rewarding loyalists," political parties can offer rewards to supporters who would vote anyway. This strategy, which does not require monitoring, has been relatively undertheorized in the literature. In recent years, scholars have made considerable advances in providing explanations for rewarding loyalists. While we acknowledge that parties in some countries do indeed engage in this strategy, we do not incorporate rewarding loyalists in the present paper.

In one explanation of rewarding loyalists, Diaz-Cayeros, Estevez, and Magaloni (forthcoming, ch. 4) argue that parties may offer particularistic benefits to core supporters during elections to sustain electoral coalitions. Their analysis, based on a formal model and data from Mexico, suggests that parties may distribute rewards to voting supporters to "prevent the erosion of partisan loyalties" over time. Unless operatives provide particularistic benefits, supporters may become swing or opposition voters during the next election.

Reciprocity may also explain why parties distribute particularistic goods to voting supporters. Parties may engage in "normative" strategies, providing goods to supporters who in turn feel a "personal obligation" to vote for a given candidate (Schaffer and Schedler 2007: 33-4; Lawson 2009). Lawson (2009) provides evidence of reciprocity in Argentina, Brazil and Mexico using survey data on clientelism. In addition, Finan and Schechter (2009), based on a field experiment and survey data, find that politicians in Paraguay are more likely to distribute rewards to reciprocal individuals, and these recipients are in turn more likely to vote for the rewarding party.

3 Combining Strategies

As the brief review above suggests, many scholars have contributed to our understanding of vote buying, turnout buying, double persuasion, negative turnout buying, and rewarding loyalists. However, a major shortcoming of the existing literature is that few studies examine how parties might in reality combine different strategies.

Empirical evidence suggests that parties do not solely engage in one strategy when distributing rewards during elections. For example, data from Argentina suggests that the Peronist party engages in both turnout buying and vote buying (Nichter 2008: 29; Dunning and Stokes 2008). The Mexico 2006 Panel Study (Lawson et al. 2007) also offers an excellent opportunity to investigate reward strategies, as it is the only panel survey to examine vote buying. Figure 2 shows the rewards received by survey respondents, categorized by each recipient's political preference vis-a-vis the party providing the good, and whether the recipient turned out in the previous presidential election.[2] This distribution of rewards in this figure provides suggestive evidence that Mexican parties do *not* solely engage in one strategy.

The key is thus not to determine whether parties unilaterally target supporters or opponents, or whether they target voters or nonvoters, but rather to examine the conditions under which they rely more heavily on one strategy versus others. Almost all existing formal papers on the topic examine only one strategy, and therefore fail

[2] Analysis based on rewards in both Wave 2 (N = 1,771) and Wave 3 (N = 1,594) of the panel survey. Includes rewards for which respondents identified candidate or party providing rewards (N = 108). Analysis excludes 7 individuals reporting more than one strategy.

Figure 2: Distribution of Rewards in Mexico's 2006 Election

		Political Preference of Recipient vis-à-vis Party Offering Goods	
		Favors Party	*Indifferent or Favors Opposition*
Turnout of Recipient in Prior Election	*Voted*	**"Rewarding Loyalists"** 20% (22)	**"Vote Buying" "Negative Turnout Buying"** 58% (63)
	Did Not Vote	**"Turnout Buying"** 7% (7)	**"Double Persuasion"** 15% (16)

to explain why a party might choose one strategy over another. For example, Stokes (2005) provides a model of vote buying, and Nichter (2008) develops a model of turnout buying. This paper contributes substantially to the literature by providing the most comprehensive model of electoral rewards developed to date.

To the best of our knowledge, only two papers—Dunning and Stokes (2008) and Morgan and Vardy (2008)—also examine the key issue of tradeoffs. Both of these studies advance scholarly research by moving in this direction. Dunning and Stokes (2008) provide an insightful analysis of the conditions under which a party might choose between vote buying and turnout buying. By contrast, Morgan and Vardy (2008) offer a cogent model that examines vote buying, turnout buying and negative turnout buying.

Despite the contributions of Dunning and Stokes (2008) and Morgan and Vardy (2008), both studies have important shortcomings. Dunning and Stokes (2008) focus narrowly on just two strategies, and thus fail to explain the majority of the reward strategies that are frequently discussed in the literature. Morgan and Vardy

(2008) do not consider double persuasion. Furthermore, Morgan and Vardy (2008) only examine contexts with either completely open ballots or completely secret ballots, neither of which corresponds to empirical evidence from many contemporary world regions. By contrast, we extend our analysis to consider imperfect ballot secrecy in Section 4.8.

Given the limitations of previous research on this topic, we now develop a model that shows how parties trade off amongst vote buying, turnout buying, and double persuasion. Key insights from the model include:

1. The party's optimal strategy is to allocate resources across *all* three strategies of vote buying, turnout buying, and double persuasion.
2. Parties will allocate relatively more resources to vote buying—and less resources to turnout buying and double persuasion—in countries with higher levels of turnout.
3. The introduction of compulsory voting will increase vote buying, while decreasing turnout buying and double persuasion.
4. An increase in the monitoring costs of a given strategy will decrease the party's usage of that strategy relative to other strategies.
5. Parties will shift away from vote buying after the introduction of the secret ballot.

A forthcoming version of this paper will also incorporate negative turnout buying into the model. For this strategy, we also provide a visual representation and intuition in Section 4.9 below.

4 Model

4.1 Setup

Consider two political parties, an incumbent machine party (M) and an opposition party (O). Each party offers a platform, x^M and x^O, respectively, on a one-dimensional ideological spectrum ranging from $\underline{X}$ to $\overline{X}$. Without loss of generality, let $x^O < x^M$, and for simplicity, assume that the parties' platforms are symmetric (that is, $x^O = -x^M$).[3]

We assume that both parties' platforms are fixed for the duration of our analysis. This simplifying assumption allows us to focus on reward targeting strategies, and also makes sense in many contexts. For example, parties may have attributes that cannot be credibly transformed in the short run, such as the personal or ideological characteristics of their leaders.

Each citizen i is defined by her political preferences x_i and voting costs c_i, where x_i and c_i are independent. The citizens' ideal points x_i are distributed over $[\underline{X}, \overline{X}]$ according to $F(x_i)$ where F has a strictly positive, continuous, and differentiable density f over $(\underline{X}, \overline{X})$, and costs of voting c_i are distributed over $[0, \overline{C}]$ according to $G(c_i)$ where G has a strictly positive, continuous, and differentiable density g over $(0, \overline{C})$. As a starting point, we assume f and g are both distributed uniformly.

A citizen who votes for the machine party receives utility:

$$U^M(x_i, c_i) = -|x^M - x_i| + d - c_i \tag{1}$$

This formulation captures the notion that the closer the citizen's ideal point to the platform of the party for which she votes, the more expressive utility she receives from casting a ballot for her favored party.[4] Because it is reasonable to believe that in some political contexts, citizens also derive utility from the very act of voting, we include the term d. Following Riker and Ordeshook (1968), this may be thought of as

[3]This assumption simplifies the algebra but qualitatively does not affect our results.

[4]We follow the assumption of previous literature on electoral reward targeting that voters receive only expressive utility, not instrumental utility, from the act of voting (Stokes 2005; Nichter 2008). Morgan and Vardy (2008) show that this assumption is justified. With a large electorate, and given a set of broadly reasonable assumptions, a citizen's probability of affecting the outcome of an election converges to zero. Therefore, it is assumed that citizens act as if only expressive utility affects their voting behavior.

a psychic benefit citizens receive from political participation, regardless of for whom they vote. It also captures the notion that some citizens may be habitual voters who turn out irrespective of their evaluations of parties' platforms. Alternatively, d can be thought of as a cost to abstention, as would occur in countries with mandatory voting rules.[5] Finally, citizens who decide to vote incur a cost c_i, which varies across individuals.

By analogous logic, the citizen who votes for the opposition party receives utility:

$$U^O(x_i, c_i) = -|x^O - x_i| + d - c_i \tag{2}$$

For simplification, we assume that voters who are indifferent cast a ballot for the machine party.

To illustrate the basic logic of the model, we initially assume that the machine has perfect information about each citizen's political preferences and voting costs, and that contracts are fully enforceable. The party consequently can act as if it observes each citizen's utility function. Furthermore, citizens will not accept rewards from the party and then vote for the opposition or stay at home on Election Day. We relax these assumptions in Section 4.7.

We assume that the objective of the machine is to maximize its net votes—the number of votes it receives minus the number of votes the opposition party receives. Given that it cannot adjust its platform, the machine's task is to win additional votes using the reward targeting strategies discussed in previous sections (see Figure 1). The machine has limited resources given by a budget level B. It must decide how to most efficiently allocate these resources across different types of citizens. In all formal analyses in this paper, we make the simplifying assumption that only the

[5] We treat d as common to all citizens, but with minor modifications our results would remain unchanged if we assumed that each citizen receives an individual-specific utility d_i from voting.

machine, not the opposition party, has the capacity to offer rewards to citizens. To facilitate exposition, we also initially assume that the machine cannot pay citizens to abstain from voting (i.e., negative turnout buying). We relax this assumption in Section 4.9.

Formally, the machine must assign a reward level $b(x_i, c_i) \in [0, B]$ to every citizen, such that $N \int_{c_i} \int_{x_i} b(x_i, c_i) f(x_i) g(c_i) dx\, dc \leq B$, where N is the total number of citizens. Limited resources means that $N \int_{c_i} \int_{x_i} \overline{b}(x_i, c_i) f(x_i) g(c_i) dx\, dc > B$, where $\overline{b}(x_i, c_i)$ is citizen i's reservation value—the payment that makes her indifferent between voting for the machine party and her next best alternative, be this voting for the opposition or abstaining. Consequently, limited resources implies that under any affordable allocation there will be citizens who receive $b(x_i, c_i) < \overline{b}(x_i, c_i)$ and are neither mobilized nor persuaded.[6]

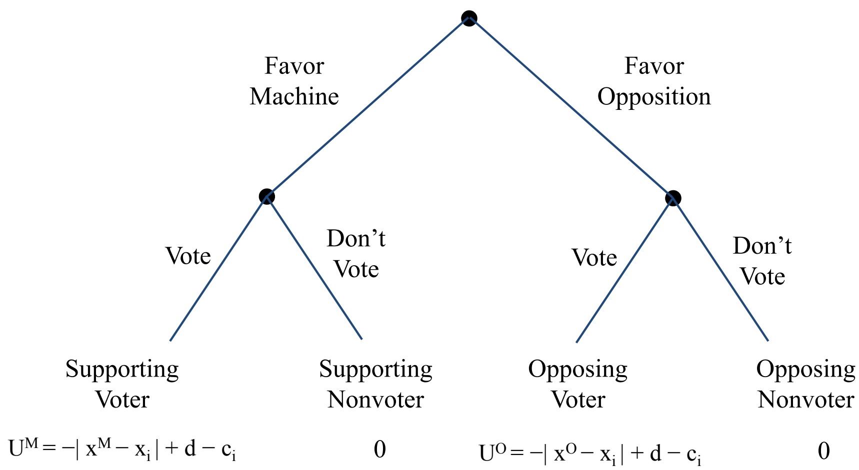
4.2 Classifying Citizens

Based on its knowledge of preferences and voting costs, the machine can analyze a citizen's decision tree (see Figure 3) and classify the population into four groups of citizens. If a citizen decides to vote, she will vote for the machine if $U^M(x_i, c_i) \geq U^O(x_i, c_i)$, or, equivalently, if $x_i \geq 0$. Citizens with political preferences $x_i \geq 0$ are thus supporters of the machine; those with political preferences $x_i < 0$ are opponents. A citizen who chooses not to vote receives no expressive utility from voting, but also incurs no voting costs. A citizen will thus choose to vote if $\max\left[U^M(x_i, c_i), U^O(x_i, c_i)\right] \geq 0$, or, equivalently, if $\max\left[-|x^M - x_i| + d, -|x^O - x_i| + d\right] \geq c_i$. To summarize, the machine can divide the population into the following groups of citizens:

- **Supporting Voters**: Citizens with political preferences $x_i \geq 0$ and for whom $-|x^M - x_i| + d \geq c_i$.
- **Supporting Nonvoters**: Citizens with political preferences $x_i \geq 0$ and for

[6]We make one additional technical assumption that $\overline{C} \geq x_i - x^M + d - c_i$. This simplifies the algebra and notation in the proofs below, but does not substantively affect our results.

Figure 3: Citizens' Decision Tree



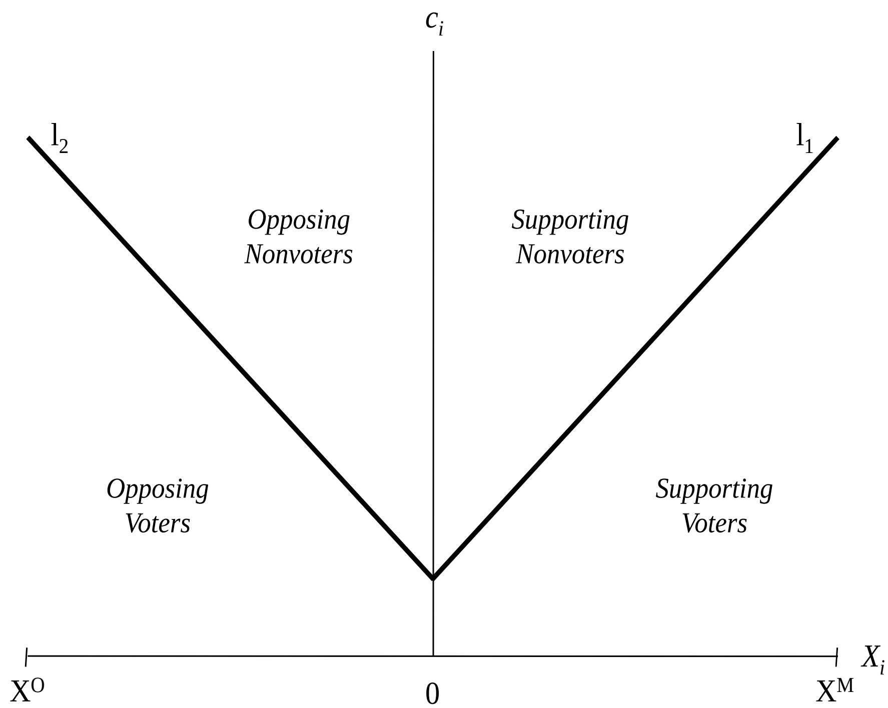
whom $-|x^M - x_i| + d < c_i$.

- **Opposing Voters**: Citizens with political preferences $x_i < 0$ and for whom $-|x^O - x_i| + d \geq c_i$.
- **Opposing Nonvoters**: Citizens with political preferences $x_i < 0$ and for whom $-|x^O - x_i| + d < c_i$.

Figure 4 presents a graphical depiction of these four groups of citizens from the machine party's viewpoint. Political preferences are represented on the horizontal axis; voting costs, on the vertical axis. The vertex lines represent citizens for whom the expressive value of voting equals voting costs, and who are thus indifferent between voting and not voting.[7] All citizen types below line l_1 vote for the machine; those below line l_2 vote for the opposition. All citizen types above l_1 and l_2 are nonvoters.

[7]Formally, these are supporters for whom $-|x^M - x_i| + d = c_i$ and opponents for whom $-|x^O - x_i| + d = c_i$. It thus follows that $l_1 = x_i - x^M + d$ and $l_2 = -x_i + x^O + d = -x_i - x^M + d$, where the second equation follows from the assumption of symmetry, $x^M = -x^O$.

Figure 4: Classifying Citizens



The vertex shape of the cutoff line between voters and nonvoters reflects the fact that a greater proportion of citizens with intense political preferences (i.e., voters for whom x_i approaches either x^M or x^O) will be voters. The reason is that, as can be seen in the utility function equations (1) and (2), they receive a high expressive utility from voting and thus are more willing to incur voting costs to support their favored party. By contrast, a smaller proportion of citizens who have weak political preferences (i.e., citizens for whom x_i approaches 0) will be voters, for they receive a lower expressive utility from voting. The inclusion of the term d, representing utility received from the act of voting independent of partisan preferences, realistically captures the notion that some indifferent voters are likely to turnout. For this reason, the tip of the vertex intercepts the vertical axis (i.e., where $x_i = 0$) at a point above the origin.[8]

4.3 Payments

In order to efficiently allocate its resources, the party must first determine the payment $\bar{b}(x_i, c_i)$ required to persuade or mobilize any given citizen type. We now examine three of the reward targeting strategies discussed in the literature (see Figure 1):[9]

Vote Buying: Vote buying targets opposing voters. These citizens have a non-negative reservation utility of $-|x^O - x_i| + d - c_i$. To persuade a opposing voter of type $t_i = (x_i, c_i)$ to switch their vote, the machine party must therefore pay $\bar{b}_{\text{VB}}(x_i, c_i)$ such that:

$$\begin{aligned} U^M(x_i, c_i) + \bar{b}_{\text{VB}}(x_i, c_i) &\geq U^O(x_i, c_i) \\ \Leftrightarrow -|x^M - x_i| + d - c_i + \bar{b}_{\text{VB}}(x_i, c_i) &\geq -|x^O - x_i| + d - c_i \end{aligned}$$

Solving for $\bar{b}_{\text{VB}}(x_i, c_i)$ and using the assumption of symmetric party platforms ($x^M = -x^O$) then yields:

$$\bar{b}_{\text{VB}}(x_i, c_i) \geq -2x_i \tag{3}$$

In the case of vote buying, the citizen already intends to vote, so the machine need only compensate her for casting a vote against her true political preferences. As shown in Inequality 3, the machine can persuade any opposing voter with a given ideal point to switch votes for the exact same price, regardless of her cost of voting.

[8] Note that at the tip of the vertex both $x_i = 0$ and $c_i = 0$, leaving a citizen with utility $d - x^M$. As long as $d \geq x^M$, some indifferent citizens will vote.

[9] We also introduce negative turnout buying in Section 4.9, once we relax the assumption that the machine cannot pay citizens to abstain from voting. This paper does not examine rewarding loyalists.

Turnout Buying: Turnout buying targets supporting nonvoters. These citizens do not intend to vote and thus have a reservation utility of zero. To mobilize a supporting nonvoter of type $t_i = (x_i, c_i)$, the party must therefore pay $\bar{b}_{\mathrm{TB}}(x_i, c_i)$ such that:

$$\begin{aligned} U^M(x_i, c_i) + \bar{b}_{\mathrm{TB}}(x_i, c_i) &\geq 0 \\ \Leftrightarrow -|x^M - x_i| + d - c_i + \bar{b}_{\mathrm{TB}}(x_i, c_i) &\geq 0 \end{aligned}$$

Solving for $\bar{b}_{\mathrm{TB}}(x_i, c_i)$ then yields:

$$\bar{b}_{\mathrm{TB}}(x_i, c_i) \geq c_i - d + x^M - x_i \tag{4}$$

In the case of turnout buying, payment is only needed to compensate a citizen for the difference between her voting costs and the positive utility she will receive from voting for the machine.

Double Persuasion: Double persuasion targets opposing nonvoters. Like supporting nonvoters, these citizens have a reservation utility of zero. To mobilize and persuade a opposing nonvoter of type $t_i = (x_i, c_i)$, the machine party must therefore pay $\bar{b}_{\mathrm{DP}}(x_i, c_i)$ such that:

$$\begin{aligned} U^M(x_i, c_i) + \bar{b}_{\mathrm{DP}}(x_i, c_i) &\geq 0 \\ \Leftrightarrow -|x^M - x_i| + d - c_i + \bar{b}_{\mathrm{DP}}(x_i, c_i) &\geq 0 \end{aligned}$$

Solving for $\bar{b}_{\mathrm{DP}}(x_i, c_i)$ then yields:

$$\bar{b}_{\mathrm{DP}}(x_i, c_i) \geq c_i - d + x^M - x_i \tag{5}$$

At first glance, this payment resembles the payment for turnout buying, but recall that for opposing nonvoters $x_i < 0$. In other words, whereas turnout buying requires the party to compensate a supporting nonvoter for the difference between her voting costs and the positive utility she receives from voting for the machine, double persuasion requires the party to compensate an opposing nonvoter both for her voting costs and for casting a vote against her political preferences.

4.4 Solving the Machine's Optimization Problem

The machine's problem is to allocate its budget B across citizen types, using the strategies of vote buying, turnout buying, and double persuasion, so as to maximize its votes relative to the opposition party's votes. An optimal allocation is thus a payment level $b(x_i, c_i)$ to each citizen type such that no other allocation of payment levels produces a (strictly) greater number of votes. Naturally, the party can choose only among affordable allocations, defined as a payment level $b(x_i, c_i) \in [0, B]$ to each citizen type such that $N \int_{c_i} \int_{x_i} b(x_i, c_i) f(x_i) g(c_i) dx\, dc \leq B$, where N is the total number of citizens.

This section considers the form the machine's optimal allocation will take:

Lemma 1: In an optimal allocation $b^*(x_i, c_i)$, the party will offer all citizens either their reservation value $\bar{b}(x_i, c_i)$ or a payment of zero. It will never overpay (i.e., pay a citizen more than her reservation value) or underpay (i.e., offer a positive payment less than a citizen's reservation value).

Proof. Recall that resource scarcity implies that for any affordable allocation, there will be citizens who are neither mobilized nor persuaded. Formally, resource scarcity means that there exists a set of positive measure, say Z, on which $b(x_i, c_i) < \bar{b}(x_i, c_i)$.

First, we will show that a necessary condition for an optimal allocation is that the party must never overpay. For contradiction, assume that $b(x_i, c_i)$ is an affordable allocation, overpays some citizens, and is an optimal allocation of B.

Formally, overpayment can be defined as the existence of a set of positive measure S such that:

$$\int_S [b(x_i, c_i) - \overline{b}(x_i, c_i)] f(x_i) g(c_i) dx\, dc > \epsilon_1 \tag{6}$$

for some $\epsilon_1 > 0$. We will demonstrate that there must exist a $b'(x_i, c_i)$ that is (i) affordable and (ii) produces more votes for the machine party than $b(x_i, c_i)$; hence, $b(x_i, c_i)$ cannot be an optimal allocation.

(i) *Affordability*: Take $(\hat{x}, \hat{c})$ to be on the interior of Z and δ_1 small enough that $\Delta(\delta_1) \equiv [\hat{x}, \hat{x} + \delta_1] \times [\hat{c}, \hat{c} + \delta_1] \subset Z$. The cost of buying all voters in $\Delta(\delta_1)$ is $N \int_{\Delta(\delta_1)} \overline{b}(x_i, c_i) f(x_i) g(c_i) dx\, dc$, which goes to zero as δ_1 goes to zero. Therefore, we can take δ_1 sufficiently small such that:

$$\int_{\Delta(\delta_1)} \overline{b}(x_i, c_i) f(x_i) g(c_i) dx\, dc < \epsilon_1 \tag{7}$$

Define $\Omega_1 \equiv [\underline{X}, \overline{X}] \times [0, \overline{C}] - (S \cup \Delta(\delta_1))$. Let $b'(x_i, c_i) = b(x_i, c_i)$ on Ω_1, and let $b'(x_i, c_i) = \overline{b}(x_i, c_i)$ on S and $\Delta(\delta_1)$. Then the affordability of $b(x_i, c_i)$ implies the affordability of $b'(x_i, c_i)$:

$$\begin{aligned}
B &\geq N \int_{c_i} \int_{x_I} b(x_i, c_i) f(x_i) g(c_i) dx\, dc \\
&= N \left[\int_{\Omega_1} b(x_i, c_i) f(x_i) g(c_i) dx\, dc + \int_S b(x_i, c_i) f(x_i) g(c_i) dx\, dc + \int_{\Delta(\delta_1)} b(x_i, c_i) f(x_i) g(c_i) dx\, dc \right] \\
&> N \left[\int_{\Omega_1} b(x_i, c_i) f(x_i) g(c_i) dx\, dc + \int_S \overline{b}(x_i, c_i) f(x_i) g(c_i) dx\, dc + \epsilon_1 \right] \\
&> N \left[\int_{\Omega_1} b(x_i, c_i) f(x_i) g(c_i) dx\, dc + \int_S \overline{b}(x_i, c_i) f(x_i) g(c_i) dx\, dc + \int_{\Delta(\delta_1)} \overline{b}(x_i, c_i) f(x_i) g(c_i) dx\, dc \right] \\
&= N \int_{c_i} \int_{x_i} b'(x_i, c_i) f(x_i) g(c_i) dx\, dc
\end{aligned}$$

where the third line follows from inequality 6 and the fourth line follows from inequality 7.

(ii) *Votes*: Note that all votes the machine receives for citizen types $(x_i, c_i) \notin \Delta(\delta_1)$ under allocation $b(x_i, c_i)$ it also receives under allocation $b'(x_i, c_i)$. But for citizen types $(x_i, c_i) \in \Delta(\delta_1)$, $b(x_i, c_i) < \bar{b}(x_i, c_i)$ while $b'(x_i, c_i) = \bar{b}(x_i, c_i)$, so all citizen types $(x_i, c_i) \in \Delta(\delta_1)$ vote for the machine under allocation $b'(x_i, c_i)$ but not under $b(x_i, c_i)$. Thus, $b'(x_i, c_i)$ buys $N \int_{\Delta(\delta_1)} f(x_i)g(c_i)dx\, dc > 0$ more voters than does $b(x_i, c_i)$.

Given that $b'(x_i, c_i)$ is both affordable and produces more votes than $b(x_i, c_i)$, $b(x_i, c_i)$ cannot be an optimal allocation.

Second, we will show that a necessary condition for an optimal allocation is that the party must never underpay. It will either pay a citizen's reservation value or offer a payment of zero. For contradiction, assume that $b(x_i, c_i)$ is affordable, offers positive payments less than reservation value to some citizens, and is an optimal allocation of B. Formally, underpayment can be defined as the existence of a set of positive measure R on which all citizens receive $b(x_i, c_i)$ such that $\bar{b}(x_i, c_i) > b(x_i, c_i) > 0$. Then under allocation $b(x_i, c_i)$, the party spends the following on citizens in set R:

$$\int_R b(x_i, c_i)f(x_i)g(c_i)dx\, dc > \epsilon_2 \tag{8}$$

for some $\epsilon_2 > 0$. We will demonstrate that there must exist a $b''(x_i, c_i)$ that is (i) affordable and (ii) produces more votes for the machine party than does $b(x_i, c_i)$; hence, $b(x_i, c_i)$ cannot be an optimal allocation.

(i) *Affordability*: Take $(\hat{x}, \hat{c})$ to be on the interior of Z and δ_2 small enough that $\Delta(\delta_2) \equiv [\hat{x}, \hat{x} + \delta_2] \times [\hat{c}, \hat{c} + \delta_2] \subset Z$. The cost of buying all voters in $\Delta(\delta_2)$ is $N \int_{\Delta(\delta_2)} \bar{b}(x_i, c_i)f(x_i)g(c_i)dx\, dc$, which goes to zero as δ_2 goes to zero. Therefore, we can take δ_2 sufficiently small such that:

$$\int_{\Delta(\delta_2)} \overline{b}(x_i, c_i) f(x_i) g(c_i) dx\, dc < \epsilon_2 \tag{9}$$

Define $\Omega_2 \equiv [\underline{X}, \overline{X}] \times [0, \overline{C}] - (R \cup \Delta(\delta_2))$. Let $b''(x_i, c_i) = b(x_i, c_i)$ on Ω_2, and let $b''(x_i, c_i) = 0$ on R and $b''(x_i, c_i) = \overline{b}(x_i, c_i)$ on $\Delta(\delta_2)$. Then the affordability of $b(x_i, c_i)$ implies the affordability of $b''(x_i, c_i)$:

$$\begin{aligned}
B &\geq N \int_{c_i} \int_{x_I} b(x_i, c_i) f(x_i) g(c_i) dx\, dc \\
&= N \left[\int_{\Omega_2} b(x_i, c_i) f(x_i) g(c_i) dx\, dc + \int_R b(x_i, c_i) f(x_i) g(c_i) dx\, dc + \int_{\Delta(\delta_2)} b(x_i, c_i) f(x_i) g(c_i) dx\, dc \right] \\
&> N \left[\int_{\Omega_2} b(x_i, c_i) f(x_i) g(c_i) dx\, dc + \epsilon_2 \right] \\
&> N \left[\int_{\Omega_2} b(x_i, c_i) f(x_i) g(c_i) dx\, dc + \int_{\Delta(\delta_2)} \overline{b}(x_i, c_i) f(x_i) g(c_i) dx\, dc \right] \\
&= N \int_{c_i} \int_{x_i} b''(x_i, c_i) f(x_i) g(c_i) dx\, dc
\end{aligned}$$

where the third line follows from inequality 8 and the fourth line from inequality 9.
(ii) *Votes*: Note that all votes the machine receives for citizen types $(x_i, c_i) \in \Omega_2$ under allocation $b(x_i, c_i)$ it also receives under allocation $b''(x_i, c_i)$. Under both allocation $b(x_i, c_i)$ and $b''(x_i, c_i)$, the party receives no votes from citizens on R because for these citizens $b(x_i, c_i) < \overline{b}(x_i, c_i)$ and $b''(x_i, c_i) < \overline{b}(x_i, c_i)$. But for citizen types $(x_i, c_i) \in \Delta(\delta_2)$, $b(x_i, c_i) < \overline{b}(x_i, c_i)$ while $b''(x_i, c_i) = \overline{b}(x_i, c_i)$, so all citizen types $(x_i, c_i) \in \Delta(\delta_2)$ vote for the machine under allocation $b''(x_i, c_i)$ but not under $b(x_i, c_i)$. Thus, $b''(x_i, c_i)$ buys $N \int_{\Delta(\delta_2)} f(x_i) g(c_i) dx\, dc > 0$ more voters than does $b(x_i, c_i)$.

Given that $b''(x_i, c_i)$ is both affordable and produces more votes than $b(x_i, c_i)$, $b(x_i, c_i)$ cannot be an optimal allocation.

The result then follows directly from the assumption of limited resources and the non-optimality of overpaying and unpaying. Resource scarcity implies that $b(x_i, c_i) \geq \overline{b}(x_i, c_i)$ for all citizens is not an affordable allocation. An optimal allocation must therefore pay those citizens for whom $b(x_i, c_i) < \overline{b}(x_i, c_i)$ nothing, while paying all other citizens exactly their reservation value.

□

The intuition behind the previous proof is straightforward. Overpayment means that the party is paying some group of citizens more than is necessary to mobilize or persuade them. Given that scarce resources implies that in any allocation, there are citizens who the party would like to buy if it had more money, the party could gain votes by reducing payments to overpaid citizens and reallocating these savings to the purchase of additional citizens. Similarly, it makes little sense for the party to offer payments to citizens below their reservation value, the amount needed to mobilize a nonvoter or switch the allegiance of an opponent. The party would end up spending money with no effect on its number of votes, so it would be better off reducing some of these underpayments to zero and reallocating the savings so as to pay at least some citizens their reservation value. Consequently, an optimal allocation will consist of two sets of citizens: those receiving their reservation value and those receiving nothing.

Lemma 2: In an optimal allocation $b^*(x_i, c_i)$, if the party buys a citizen (x_i, c_i), then it will buy all cheaper citizens.

Proof. [NOTE: This proof is not fully complete and will be revised.] Define $M(b)$ to be the set of citizens who vote for the machine party given the payment allocation $b(x_i, c_i)$: $M(b) \equiv \{(x_i, c_i) : b(x_i, c_i) \geq \overline{b}(x_i, c_i)\}$. Let $(\hat{x}, \hat{c})$ be any point on the interior of $M(b)$. For contradiction, assume $b(x_i, c_i)$ is an affordable, optimal allocation in which the party does not buy all citizens who are cheaper than $(\hat{x}, \hat{c})$. Formally, there exists a set Q with positive measure, such that $b(x_i, c_i) < \overline{b}(x_i, c_i) < \overline{b}(\hat{x}, \hat{c})$

for all $(x_i, c_i) \in Q$. We will show that there must exist a set $b'(x_i, c_i)$ that produces the same number votes for the machine as $b(x_i, c_i)$ but for less money; hence $b(x_i, c_i)$ cannot be optimal.

Take $\Delta(\delta) \equiv [\hat{x}, \hat{x}+\delta] \times [\hat{c}, \hat{c}+\delta]$. Since $(\hat{x}, \hat{c})$ is on the in the interior of $M(b)$, there must exist δ sufficiently small that $\Delta(\delta) \subset M(b)$. Let (x_i, c_i) be any point in Q and select μ sufficiently small such that $\Delta(\mu) \equiv [x_i, x_i + \mu] \times [c_i, c_i + \mu] \subset Q$.

By the continuity of $f(x_i)$ and $g(c_i)$ there exists a $\delta_0 < \delta$ and $\mu_0 < \mu$ such that:

$$\int_{\Delta(\delta_0)} f(x_i)g(c_i)dx\,dc = \int_{\Delta(\mu_0)} f(x_i)g(c_i)dx\,dc$$

That is $\Delta(\delta_0)$ and $\Delta(\mu_0)$ contain the same number of voters. Because the voters in $\Delta(\mu_0)$ are cheaper than those in $\Delta(\delta_0)$, it follows that:

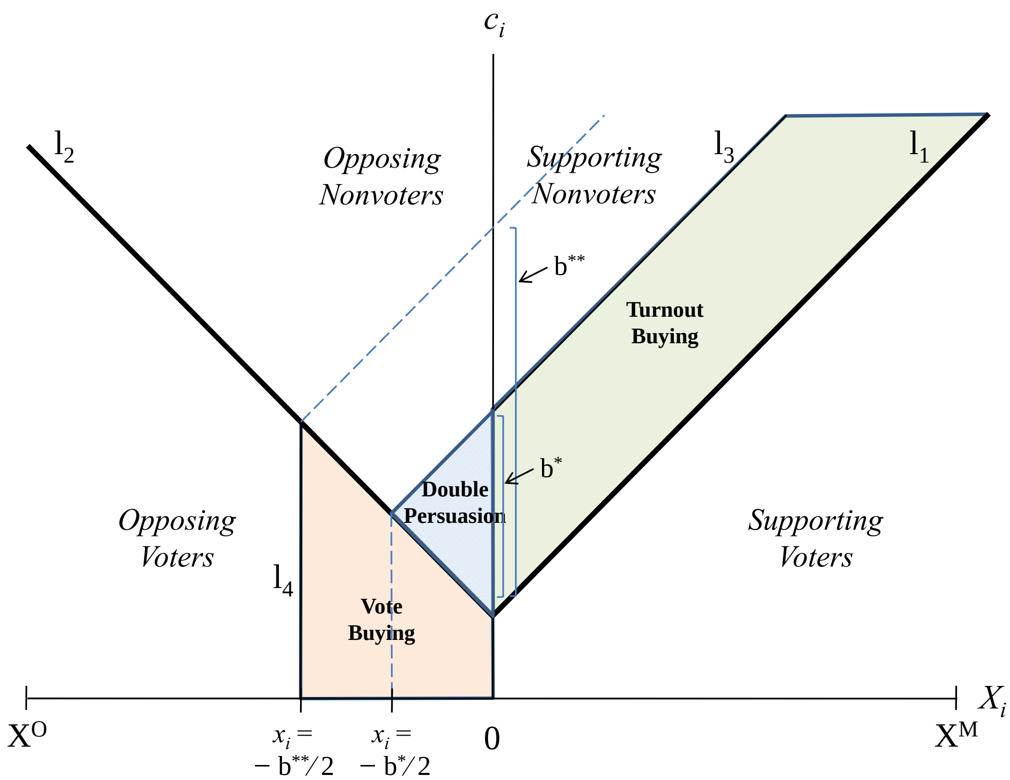
$$\int_{\Delta(\delta_0)} b(x_i, c_i)f(x_i)g(c_i)dx\,dc \geq \int_{\Delta(\delta_0)} \overline{b}(x_i, c_i)f(x_i)g(c_i)dx\,dc > \int_{\Delta(\mu_0)} \overline{b}(x_i, c_i)f(x_i)g(c_i)dx\,dc$$
$$\Leftrightarrow \int_{\Delta(\delta_0)} b(x_i, c_i)f(x_i)g(c_i)dx\,dc - \int_{\Delta(\mu_0)} \overline{b}(x_i, c_i)f(x_i)g(c_i)dx\,dc \equiv \theta > 0$$

Define $\Omega \equiv [\underline{X}, \overline{X}] \times [0, \overline{C}] - (\Delta(\delta_0) \cup \Delta(\mu_0))$ and let $b'(x_i, c_i) = b(x_i, c_i)$ on Ω, $b'(x_i, c_i) = \overline{b}(x_i, c_i)$ on $\Delta(\mu_0)$, and $b'(x_i, c_i) = 0$ on $\Delta(\delta_0)$. Then $b'(x_i, c_i)$ produces the same number of votes as $b(x_i, c_i)$ for less money; $b(x_i, c_i)$ therefore cannot be an optimal allocation of B.

$\square$

The logic of the previous proof is similar to the reason why the party optimally should not offer overpayment or underpayments. In order to maximize its number of net votes, the party should allocate funds to the cheapest possible citizens. Otherwise, it would be possible to reduce payments to more expensive citizens, reallocate these resources to cheaper citizens, and obtain more votes for the same price.

Figure 5: Targeting Citizen Types



Combining the results of Lemmas 1 and 2, as well as the payment equations from section 4.3, it is now possible to characterize the party's optimal targeting strategy.

Proposition 1: The party's optimal strategy will be to offer a payment b^* to all nonvoters in the set $t^* = \{(x^*, c^*) : c^* = x^* - x^M + d + b^*, x^* \in [\frac{b^*}{2}, \overline{X}]\}$ and $b^{**} = 2b^*$

to all opposition voters in the set $t^{**} = \{(x^{**}, c^{**}) : x^{**} = -\frac{b^{**}}{2}\}$. This is the set of nonvoters along line l_3 and the voters along line l_4 in Figure 5. All nonvoters above l_3 and to the left of l_4 will receive $b(x_i, c_i) = 0$. All nonvoters below l_3 and all opposition voters to the right of l_4 will receive payments in accordance with payment equations (3), (4), and (5).

Proof. Let b^*_{VB}, b^*_{TB}, and b^*_{DP} be the upper bounds on the payments the party makes to citizens through vote buying, turnout buying, or double persuasion, respectively, in an optimal allocation of its budget B. By payment equation (3), we know that the party must offer $b_{\mathrm{VB}} \geq -2x_i$ to vote buy any given citizen (x_i, c_i). By Lemma 1, we know that in an optimal allocation the party will never overpay or underpay, so it will offer any opposing voter who it vote buys $b_{\mathrm{VB}} = -2x_i$ or 0. Then for payment b^*_{VB} all opposing voters of type $t^*_{\mathrm{VB}} = \{(x^*_{\mathrm{VB}}, c^*_{\mathrm{VB}}) : x^*_{\mathrm{VB}} = -\frac{b^*_{\mathrm{VB}}}{2}\}$ can be bought and to avoid under or overpayment, all opposing voters in this set must receive the same payment. Finally, by Lemma 2, if the party buys opposing voters of type $(x^*_{\mathrm{VB}}, c^*_{\mathrm{VB}})$, in an optimal allocation of B it must also buy all cheaper opposing voters.

By analogous logic, according to payment equations (4) and (5), the machine party must offer $b^*_{\mathrm{DP}} \geq c_i - d + x^M - x_i$ to any non-voting opponent it wants to double persuade and $b^*_{\mathrm{TB}} \geq c_i - d + x^M - x_i$ to any non-voting supporter it wants to turnout buy; by Lemma 1 it will never overpay or underpay so it will offer exactly these amounts or a payment of zero to all non-voting citizens. Then for payment b^*_{DP} all non-voting opponents in the set $t^*_{\mathrm{DP}} = \{(x^*_{\mathrm{DP}}, c^*_{\mathrm{DP}}) : c^*_{\mathrm{DP}} = x^*_{\mathrm{DP}} - x^M + d + b^*_{\mathrm{DP}}, x^* \in [-\frac{b^*}{2}, 0\}$ can be bought and to avoid over or under payment, all citizens in this set must receive the same payment. Likewise, for b^*_{TB} all non-voting supporters in the set $t^*_{\mathrm{TB}} = \{(x^*_{\mathrm{Tb}}, c^*_{\mathrm{TB}}) : c^*_{\mathrm{TB}} = x^*_{\mathrm{TB}} - x^M + d + b^*_{\mathrm{TB}}, x^* \in [0, \overline{X}]\}$ can be bought and to avoid over or underpayment, all citizens in this set must receive the same payment. Again, by Lemma 2, if the party buys citizens of type $(x^*_{\mathrm{DP}}, c^*_{\mathrm{DP}})$ or $(x^*_{\mathrm{TB}}, c^*_{\mathrm{TB}})$, then in an optimal allocation of B it must buy all cheaper nonvoters.

Finally, we will prove that $b^*_{\mathrm{TB}} = b^*_{\mathrm{DP}} = b^*$ and $b^*_{\mathrm{VB}} = b^{**} = 2b^*$. Let N be the total number of citizens, V^M be the number of citizens who vote for the machine party, and V^O be the number of citizens who vote the opposition. For notational simplicity let $r = d - x^M$. Define:

$$V^M = VB + DP + TB + S \tag{10}$$

where VB, the number of opposing voters who receive payments (i.e., the extent of vote buying), is:

$$VB = N \int_0^{-x_i + r} \int_{-\frac{b^*_{\mathrm{VB}}}{2}}^0 f(x_i) g(c_i) dx\, dc \tag{11}$$

DP, the number of opposing nonvoters who receive payments (i.e., the extent of double persuasion), is:

$$DP = N \int_{-x_i + r}^{x_i + r + b^*_{\mathrm{DP}}} \int_{-\frac{b^*_{\mathrm{DP}}}{2}}^0 f(x_i) g(c_i) dx\, dc \tag{12}$$

TB, the number of supporting nonvoters who receive payments (i.e., the extent of turnout buying), is:

$$TB = N \int_{x_i + r}^{x_i + r + b^*_{\mathrm{TB}}} \int_0^{\overline{X}} f(x_i) g(c_i) dx\, dc \tag{13}$$

and S, the number of supporting voters (who vote for the machine without receiving payments), is:

$$S = N \int_0^{x_i + r} \int_0^{\overline{X}} f(x_i) g(c_i) dx\, dc \tag{14}$$

Finally, V^O, the number of votes the opposition party will receive after the machine makes payments, is:

$$V^O = N \int_0^{-x_i+r} \int_{\underline{X}}^{-\frac{b^*_{\mathrm{VB}}}{2}} f(x_i)g(c_i)dx\,dc \tag{15}$$

The machine party's expenditures on targeted rewards can be calculated in a similar manner. Define total expenditures E as:

$$E = E_{\mathrm{VB}} + E_{\mathrm{DP}} + E_{\mathrm{TB}} \tag{16}$$

where E_{VB}, expenditures on payments to opposing voters (vote-buying payments), are:[10]

$$E_{\mathrm{VB}} = N \int_0^{-x_i+r} \int_{-\frac{b^*_{\mathrm{VB}}}{2}}^{0} -(2x_i)f(x_i)g(c_i)dx\,dc \tag{17}$$

E_{DP}, expenditures on payments to opposing nonvoters (double-persuasion payments), are:

$$E_{\mathrm{DP}} = N \int_{-x_i+r}^{x_i+r+b^*_{\mathrm{DP}}} \int_{-\frac{b^*_{\mathrm{DP}}}{2}}^{0} (c_i - r - x_i)f(x_i)g(c_i)dx\,dc \tag{18}$$

and E_{TB}, expenditures on payments to supporting nonvoters (turnout-buying payments), are:

$$E_{\mathrm{TB}} = N \int_{x_i+r}^{x_i+r+b^*_{\mathrm{TB}}} \int_0^{\overline{X}} (c_i - r - x_i)f(x_i)g(c_i)dx\,dc \tag{19}$$

Combining these equations, we can write the machine's constrained optimization problem, where λ is the Lagrangian multiplier:

$$\max_{b^*_{\mathrm{TB}},\, b^*_{\mathrm{DP}},\, b^*_{\mathrm{VB}}} V^M - V^O - \lambda(E - B) \tag{20}$$

Solving this problem yields the following first-order conditions:

[10]Recall that $-2x_i > 0$ for opponents.

$$\frac{\partial VB}{\partial b^*_{\text{VB}}} - \frac{\partial V^O}{\partial b^*_{\text{VB}}} = 2\frac{\partial VB}{\partial b^*_{\text{VB}}} = \lambda\frac{\partial E_{\text{VB}}}{\partial b^*_{\text{VB}}} \tag{21}$$

$$\frac{\partial DP}{\partial b^*_{\text{DP}}} = \lambda\frac{\partial E_{\text{DP}}}{\partial b^*_{\text{DP}}} \tag{22}$$

$$\frac{\partial TB}{\partial b^*_{\text{TB}}} = \lambda\frac{\partial E_{\text{TB}}}{\partial b^*_{\text{TB}}} \tag{23}$$

where the first equality in the first FOC follows from $\frac{\partial VB}{\partial b^*_{\text{VB}}} = -\frac{\partial V^O}{\partial b^*_{\text{VB}}}$.

Define $\Gamma = \frac{N}{(\overline{X}-\underline{X})\overline{C}}$. Then under the assumption that $f(x_i)$ and $g(c_i)$ are distributed uniformly, the first order conditions above become:

$$\frac{\Gamma}{4}\left[2b^*_{\text{VB}} + 4d\right] = \lambda\frac{\Gamma}{4}\left[(b^*_{\text{VB}})^2 + 2b^*_{\text{VB}}d\right] \tag{24}$$

$$\frac{\Gamma}{2}b^*_{\text{DP}} = \lambda\frac{\Gamma}{2}(b^*_{\text{DP}})^2 \tag{25}$$

$$\Gamma\overline{X} = \lambda\Gamma\overline{X}b^*_{\text{TB}} \tag{26}$$

Solving all first order conditions for λ yields the results: $b^*_{\text{TB}} = b^*_{\text{DP}} = b^*$, and $b^*_{\text{VB}} = 2b^*$.

□

Intuitively, the machine wants to buy as many votes as possible given its limited resources. According to the payment equations (4) and (5), all nonvoters with a pair of preferences and net voting costs (x_i^*, c_i^*) such that $b^* \geq c_i^* - d + x^M - x_i$ would be willing to vote for the machine at this price. The party, however, never would pay more than necessary (by Lemma 1), and therefore would only be willing to offer such a price to the set of citizens for which this inequality binds. This is the set of citizens along the line l_3 in Figure 5, which correspondingly has slope x_i and intersects the vertical axis at $d - x^M + b^*$. Naturally, if the party is willing to pay b^* to these citizens, it must be willing to buy cheaper nonvoters given Lemma 2. The area below l_3 and above l_1 thus represents citizen types whose turnout will be bought, and the area below l_3 and above l_2 represents citizen types who will be double persuaded. Per Lemma 1, any non-voting citizen type who would require a higher price than $b^*(x_i^*, c_i^*)$ to turn out and support the party will receive no payment.

When the party uses turnout buying or double persuasion, it mobilizes one additional voter. At the margin, it logically should be willing to pay the same price for a non-voting supporter and a non-voting opponent. However, when it relies on vote buying, it receives an additional vote *and* takes away a vote from the opposition. At the margin, the party should thus be willing to spend twice as much on vote buying than on strategies that mobilize nonvoters (the proof to proposition 1 confirms this intuition): $b^*_{\mathrm{VB}} = b^{**} = 2b^*$. According to the payment equation (3), all voting opponents with preferences x_i^{**} such that $b^{**} \geq -2x_i^{**}$ would be willing to switch their vote in exchange for this payment. The party would again not be willing to pay more than necessary, and therefore would offer such a payment only to citizen types for which this inequality binds. This is the set of citizen types along the vertical component of line l_4 where $x_i = -\frac{b^{**}}{2}$. If the party is willing to pay b^{**} to persuade an opposing voter, it must be willing to buy cheaper opponents. The area to the right of l_4 and under l_2 thus represents citizen types who will receive vote-buying payments. Per Lemma 1, any opposing voter who would require a higher price than $b^{**}(x^{**}, c^{**})$ to switch their votes will receive no payment.

4.5 Analysis

A notable finding emerges directly from the form of the party's optimal allocation strategy:

Proposition 2: The party's optimal strategy will allocate resources across *all* three strategies of vote buying, turnout buying, and double persuasion. It will never employ: (1) exclusively vote buying, (2) exclusively turnout buying, (3) exclusively double persuasion, or (4) any combination that includes only two of these three strategies.

Proof. This result follows directly from the proof for proposition 1. The conclusion $b^*_{\text{TB}} = b^*_{\text{DP}} = b^*$ and $b^*_{\text{VB}} = 2b^*$, implies $b^*_{\text{TB}} > 0 \iff b^*_{\text{DP}} > 0 \iff b^*_{\text{VB}} > 0$. It then follows from equations (11), (12), and (13) that $VB > 0 \iff TB > 0 \iff DP > 0$. □

Proposition 2 provides an analytical foundation for why parties combine strategies. Rather than relying exclusively on particular reward targeting strategies, evidence discussed earlier suggests that parties simultaneously draw on a portfolio of strategies. Our model illuminates the logic of this multi-strategy approach. Parties seek to target the cheapest citizens available, which requires both mobilization and persuasion. In addition, Proposition 2 points to a somewhat non-intuitive result: the machine will rely on the strategy of double persuasion, even though this requires a dual effort to mobilize and persuade. While other scholars have labeled double persuasion a "perverse strategy" (Dunning and Stokes 2008), our model indicates why this is not the case. Once the party devotes resources to buying the cheapest opposing voters (vote buying) and supporting nonvoters (turnout buying), there will be a group of opposing nonvoters who can be bought more cheaply than opposing voters with stronger ideological preferences or supporting nonvoters with higher voting costs.

Although the party employs a mix of strategies, it does not necessarily rely on all strategies equally. The model offers insights into the conditions under which the party favors one strategy over another and allows us to explore the potential impact of institutional changes. We now consider the example of how the implementation of compulsory voting affects the types of reward strategies parties employ.

4.6 Impact of Compulsory Voting

We now examine how compulsory voting is expected to affect reward strategies. Voting is compulsory in at least 30 countries, including Argentina, Belgium, Italy, Turkey, and Uruguay (IDEA 2009). Compulsory voting is the "strongest of all the institutional factors" affecting turnout (Lijphart 1997: 8). Influential cross-national studies by Powell (1980) and Jackman (1987) find that compulsory voting is associated with a 10 and 13 percent higher rate of turnout, respectively. Within-country comparisons provide even more compelling evidence; for example, Australia experienced an increase of over 28 percent in turnout after introducing compulsory voting in 1924 (Hirczy 1994).

Given that the introduction of compulsory has dramatic impact on the level of turnout, how would this institutional change shift the strategies of vote buying, turnout buying, and double persuasion? To explore this question, we examine the imposition of a penalty for not showing up at the polls. In countries with compulsory voting, penalties vary substantially, including fines of AU$20 ($14) in Australia and difficulty obtaining official documents in Brazil and Greece (Australian Electoral Commission 2009; IDEA 2009). As discussed in Section 4.1, the term d in our model captures the cost of abstention, so the imposition of a penalty increases d. Analysis of comparative statics yields the following proposition:

Proposition 3: As turnout levels increase, as in the case of the introduction of compulsory voting rules, the machine party relies more heavily on vote buying relative to turnout buying and double persuasion ($\frac{dVB}{dd} > 0$, $\frac{dTB}{dd} < 0$, $\frac{dDP}{dd} < 0$).

Proof. To examine comparative statics, substitute $b^{**} = 2b^*$ into the budget constraint $E = B$, yielding (for the uniform distribution case):

$$B = \frac{\Gamma}{6}\left[5(b^*)^3 + 6(b^*)^2 d + 3(b^*)^2 \overline{X}\right] \tag{27}$$

Likewise, substituting $b^* = \frac{1}{2}b^{**}$ into the budget constraint $E = B$ yields:

$$B = \frac{\Gamma}{48}\left[5(b^{**})^3 + 12(b^{**})^2 d + 6(b^{**})^2\overline{X}\right] \tag{28}$$

By implicit differentiation of the budget constraint equations above:

$$\begin{aligned}
\frac{\partial b^*}{\partial d} &= \frac{-2b^*}{4d + 2\overline{X} + 5b^*} < 0 \\
\frac{\partial b^{**}}{\partial d} &= \frac{-4b^{**}}{8d + 4\overline{X} + 5b^{**}} < 0
\end{aligned}$$

Comparative statics then follow:

$$\begin{aligned}
\frac{dVB}{dd} &= \frac{\partial VB}{\partial d} + \frac{\partial VB}{\partial b^{**}}\frac{\partial b^{**}}{\partial d} = \frac{\Gamma}{4}\left[2b^{**} + (2d + b^{**})\frac{\partial b^{**}}{\partial d}\right] \\
&= \frac{\Gamma}{4}\left[2b^{**} - 2b^{**}\left(\frac{4d + 2b^{**}}{8d + 4\overline{X} + 5b^{**}}\right)\right] > 0 \\
\frac{dTB}{dd} &= \frac{\partial TB}{\partial d} + \frac{\partial TB}{\partial b^*}\frac{\partial b^*}{\partial d} = \Gamma\overline{X}\frac{\partial b^*}{\partial d} < 0 \\
\frac{dDP}{dd} &= \frac{\partial DP}{\partial d} + \frac{\partial DP}{\partial b^*}\frac{\partial b^*}{\partial d} = \Gamma\frac{b^*}{2}\frac{\partial b^*}{\partial d} < 0
\end{aligned}$$

□

Thus, the comparative statics predict that with the introduction of compulsory voting, the machine relies more heavily on vote buying relative to turnout buying and double persuasion. Intuitively, with this institutional change, turnout increases because citizens must now weigh the costs of voting against sanctions from abstaining. Graphically, this shifts the vertex composed of $l1$ and $l2$ in Figure 5 upward. There are now more weak opposing voters clustered along the vertical axis. These opposing voters are among the cheapest citizens to buy. The cost of mobilizing supporters, meanwhile, remains unchanged. The party accordingly shifts resources from mobilization to persuasion.

4.7 Transaction Costs

Up to this point, we have assumed that the party has perfect information about citizens' types, and that contracts are fully enforceable. Here we examine a more realistic setting in which the enforceability assumption is relaxed. To motivate this analysis, consider the possibility that an opposing voter can claim a payment and then defect, voting for the opposition party. If she defects, with probability $q_{\mathrm{VB}} \in [0, 1]$ the party observes this defection and she does not receive payment. The condition to prevent defection is thus:

$$\begin{aligned} U^M(x_i, c_i) + \tilde{b}_{\mathrm{VB}}(x_i, c_i) &\geq q_{\mathrm{VB}} U^O(x_i, c_i) + (1 - q_{\mathrm{VB}})[U^O(x_i, c_i) + \tilde{b}_{\mathrm{VB}}(x_i, c_i)] \\ \iff \tilde{b}_{\mathrm{VB}}(x_i, c_i) &\geq \frac{1}{q_{\mathrm{VB}}}[-(2x_i)] = \frac{1}{q_{\mathrm{VB}}}\, \bar{b}_{\mathrm{VB}}(x_i, c_i) \end{aligned} \tag{29}$$

Here, $\tilde{b}_{\mathrm{VB}}$ represents the payment required for vote buying when contracts are not fully enforceable, whereas $\bar{b}_{\mathrm{VB}}$ represents the payment required for vote buying with perfect enforcement. In other words, without fully enforceable contracts, the party must pay a premium $\frac{1}{q_{\mathrm{VB}}}$ on every dollar spent on vote buying in order to prevent defection.[11] Similar logic applies to the use of double persuasion and turnout buying strategies, except for that it is reasonable to assume that the challenges of monitoring turnout are different than the challenges of monitoring *how* a citizen votes. We capture this notion by allowing the party's probability of observing defection to differ for each of the types of contracts (i.e., $q_{\mathrm{VB}} \neq q_{\mathrm{TB}} \neq q_{\mathrm{DP}}$). It then

[11] An alternative setup would be to assume that the citizens and party are playing a repeated prisoner's dilemma game, as in Stokes (2005) and Nichter (2008). Repeated play makes prevention of defection possible, but again the party must pay a premium such that the citizen receives a total payoff above her reservation utility. The upshot is the same as in the simpler setup presented here.

follows by analogous analysis to equation (29) that $\tilde{b}_{\text{DP}}(x_i, c_i) = \frac{1}{q_{\text{DP}}}\,\bar{b}_{\text{DP}}(x_i, c_i)$ and $\tilde{b}_{\text{TB}}(x_i, c_i) = \frac{1}{q_{\text{TB}}}\,\bar{b}_{\text{TB}}(x_i, c_i)$.

Let $\alpha = \frac{1}{q_{\text{VB}}}$,$\gamma = \frac{1}{q_{\text{DP}}}$, and $\beta = \frac{1}{q_{\text{TB}}}$. Then, taking into account transaction costs, we can re-characterize the party's optimal strategy:

Proposition 4: Given transaction costs, the party's optimal strategy will be to:

(i) offer a payment $\tilde{b}^*_{\text{TB}}$ to all non-voting supporters in the set $\tilde{t}^*_{\text{TB}} = \{(\tilde{x}^*, \tilde{c}^*) : \tilde{c}^* = \tilde{x}^* - x^M + d + \tilde{b}^*_{\text{TB}}, \tilde{x}^* \in [0, \overline{X}]\}$; offer a payment $\tilde{b}^*_{\text{DP}}$ to all non-voting opponents in the set $\tilde{t}^*_{\text{DP}} = \{(\tilde{x}^*, \tilde{c}^*) : \tilde{c}^* = \tilde{x}^* - x^M + d + \tilde{b}^*_{\text{DP}}, \tilde{x}^* \in [\frac{\tilde{b}^*_{\text{DP}}}{2}, 0]\}$; and offer $\tilde{b}^*_{\text{VB}}$ to all opposition voters in the set $\tilde{t}^*_{\text{VB}} = \{(\tilde{x}^*, \tilde{c}^*) : \tilde{x}^*_{\text{VB}} = -\frac{\tilde{b}^*_{\text{VB}}}{2}\}$.

(ii) offer $\tilde{b}_{\text{VB}}(x_i, c_i)$ to all opposing voters who can be persuaded for less than $\tilde{b}^*_{\text{VB}}$; offer $\tilde{b}_{\text{DP}}(x_i, c_i)$ to all opposing nonvoters who can be mobilized and persuaded for less than $\tilde{b}^*_{\text{DP}}$; and offer $\tilde{b}_{\text{TB}}(x_i, c_i)$ to all non-voting supporters who can be mobilized for less than $\tilde{b}^*_{\text{TB}}$.

(iii) offer a zero payment to opposition voters for whom it would cost more than $\tilde{b}^*_{\text{VB}}$ to mobilize; to opposition nonvoters for whom it would cost more than $\tilde{b}^*_{\text{DP}}$ to persuade and mobilize; and to non-voting supporters for whom it would cost more than $\tilde{b}^*_{\text{TB}}$ to mobilize.

(iv) set $\alpha\tilde{b}^*_{\text{VB}} = 2\gamma\tilde{b}^*_{\text{DP}} = 2\beta\tilde{b}^*_{\text{TB}}$.

Proof. The proof is identical to the proof of Lemmas 1 and 2 and Proposition 1, except for replace b^*_{VB}, b^*_{TB}, and b^*_{DP} with $\tilde{b}^*_{\text{VB}}$, $\tilde{b}^*_{\text{TB}}$, and $\tilde{b}^*_{\text{DP}}$, and let $\tilde{E}_{\text{VB}} = \alpha E_{\text{VB}}$, $\tilde{E}_{\text{TB}} = \beta E_{\text{TB}}$, and $\tilde{E}_{\text{DP}} = \gamma E_{\text{DP}}$. The FOCs then become:

$$\frac{\partial \tilde{VB}}{\partial \tilde{b}^*_{\text{VB}}} - \frac{\partial \tilde{V}^O}{\partial \tilde{b}^*_{\text{VB}}} = 2\frac{\partial \tilde{VB}}{\partial \tilde{b}^*_{\text{VB}}} = \alpha\lambda \frac{\partial \tilde{E}_{\text{VB}}}{\partial \tilde{b}^*_{\text{VB}}} \tag{30}$$

$$\frac{\partial \tilde{DP}}{\partial \tilde{b}^*_{\text{DP}}} = \gamma\lambda \frac{\partial \tilde{E}_{\text{DP}}}{\partial \tilde{b}^*_{\text{DP}}} \tag{31}$$

$$\frac{\partial \tilde{TB}}{\partial \tilde{b}^*_{\text{TB}}} = \beta\lambda \frac{\partial \tilde{E}_{\text{TB}}}{\partial \tilde{b}^*_{\text{TB}}} \tag{32}$$

Substituting in the uniform distribution and solving for λ gives the result: $\alpha\tilde{b}^*_{\text{VB}} = 2\beta\tilde{b}^*_{\text{TB}} = 2\gamma\tilde{b}^*_{\text{DP}}$

□

The intuition is similar to the logic expressed in section 2.4. For a strategy to be optimal, the marginal cost of the last citizen targeted for turnout buying must be equal to the marginal cost of the last citizen targeted for double persuasion, given that they each produce a marginal benefit of one vote. The marginal cost of the last citizen targeted for vote buying, meanwhile, must equal twice the marginal cost of the last citizens to be turnout bought or double persuaded, given that vote buying produces an additional vote and takes a vote away from the opposing party. These marginal costs, in turn now depend on the monitoring costs of each strategy.

Adding transaction costs to the analysis yields further insights into factors influencing the party's choice of different strategies. Next, we consider the example of introducing the secret ballot, which raises the monitoring costs of vote buying relative to other strategies:

4.8 Impact of Ballot Secrecy

We now examine the effect of ballot secrecy on parties' reward strategies. Before the introduction of the secret ballot, vote choices could be monitored relatively easily. Open voting thus made it considerably cheaper for parties to ensure compliance when offering rewards to voters in exchange for switching vote choices. A major argument in favor of introducing the secret ballot was the expectation that it would reduce vote buying, which was relatively common with open voting (cf Orr 2006: 307; Lehoucq 2002:6). For example, a US newspaper in 1888 commented that "if the act of voting were performed in secret, no bribed voter could or would be trusted to carry out his bargain when left to himself" (cf Campbell 2005: 97).

What are our model's predictions for the introduction of the secret ballot? To explore this question, we examine the effect of an increase in the monitoring cost of vote buying (α) vis-a-vis the monitoring costs of turnout buying (β). Analysis of comparative statics yields the following proposition:

Proposition 5: An increase in the monitoring costs of a given strategy (e.g., vote buying), as in the case of the introduction of secret balloting, will reduce the party's reliance on this strategy relative to its use of other strategies ($\frac{\partial \tilde{VB}}{\partial \alpha} < 0$). Similarly, an increase in the monitoring costs of other strategies (e.g., turnout buying) will increase the party's reliance on a given strategy (e.g., vote buying) relative to its use of other strategies ($\frac{\partial \tilde{VB}}{\partial \beta} > 0$).

Proof. We demonstrate the effects of changing monitoring costs (α and β) on the extent of vote buying. The proofs for the effects of changing monitoring costs on turnout buying or double persuasion are analogous.

From the FOCs in the proof for proposition 4, substitute $\tilde{b}^*_{\text{TB}} = \frac{\alpha}{2\beta}\tilde{b}^*_{\text{VB}}$ and $\tilde{b}^*_{\text{DP}} = \frac{\alpha}{2\gamma}b^*_{\text{VB}}$ into the budget equation $\tilde{E} = \tilde{B}$, yielding:

$$B = \frac{\Gamma}{48}\left[4\alpha(\tilde{b}^*_{VB})^3 + 12\alpha d(\tilde{b}^*_{\text{VB}})^2 + \frac{6\alpha^2\overline{X}}{\beta}(\tilde{b}^*_{\text{VB}})^2 + \frac{\alpha^3}{\gamma^2}(\tilde{b}^*_{VB})^3\right]$$

Implicit differentiation of the above budget equation yields:

$$\frac{\partial \tilde{b}^*_{\text{VB}}}{\partial \alpha} = -\frac{\tilde{b}^*_{\text{VB}}(12\alpha\gamma^2\overline{X} + 12d\beta\gamma^2 + 3\alpha^2\beta\tilde{b}^*_{\text{VB}} + 4\beta\gamma^2\tilde{b}^*_{\text{VB}})}{3\alpha(4\alpha\gamma^2\overline{X} + 8d\beta\gamma^2 + \alpha^2\beta\tilde{b}^*_{\text{VB}} + 4\beta\gamma^2\tilde{b}^*_{\text{VB}})} < 0$$

$$\frac{\partial \tilde{b}^*_{\text{VB}}}{\partial \beta} = \frac{2\alpha\gamma^2\overline{X}\tilde{b}^*_{\text{VB}}}{\beta(4\alpha\gamma^2\overline{X} + 8d\beta\gamma^2 + \beta\alpha^2\tilde{b}^*_{\text{VB}} + 4\beta\gamma^2\tilde{b}^*_{\text{VB}})} > 0$$

Comparative statics then follow:

$$\frac{\partial \tilde{VB}}{\partial \alpha} = \frac{\partial \tilde{VB}}{\partial \tilde{b}^*_{\text{VB}}}\frac{\partial \tilde{b}^*_{\text{VB}}}{\partial \alpha} = \frac{\Gamma}{4}\left[(2d + \tilde{b}^*_{\text{VB}})\frac{\partial \tilde{b}^*_{\text{VB}}}{\partial \alpha}\right] < 0$$

$$\frac{\partial \tilde{VB}}{\partial \beta} = \frac{\partial \tilde{VB}}{\partial \tilde{b}^*_{\text{VB}}}\frac{\partial \tilde{b}^*_{\text{VB}}}{\partial \beta} = \frac{\Gamma}{4}\left[(2d + \tilde{b}^*_{\text{VB}})\frac{\partial \tilde{b}^*_{\text{VB}}}{\partial \beta}\right] > 0$$

□

The logic behind this proposition is clear. The introduction of a secret ballot dramatically increases the costs of monitoring *how* citizens vote, but it leaves the costs of monitoring *whether* they vote unchanged. Because the cost of vote buying relative to other strategies increases, the party shifts resources away from vote buying. More broadly, when one strategy is relatively cheaper than other strategies, we expect to see parties focus more resources on this less expensive strategy.

The model's predictions for ballot secrecy are consistent with the empirical literature on the topic. The broad consensus is that the prevalence of vote buying does indeed fall after the introduction of the secret ballot (e.g., Cox 2006: 5; Hasen 2000: 1328; Heckelman 1995: 107). As Hasen (2000: 1328) explains, "with the rise of the secret ballot and the concomitant increase in the cost of verifying that vote buyers were getting what they paid for, vote buying almost certainly has declined." Of course, while vote buying has decreased with the secret ballot, it has not disappeared altogether. As discussed in Section 2.1, parties have developed (albeit more costly) ways of monitoring vote choices despite secret ballot laws (see also Brusco, Nazareno and Stokes 2004; Stokes 2005). The finding that vote buying continues to coexist with other strategies is consistent with Proposition 2, which suggests that parties' optimal strategy will be to allocate resources across *all* three strategies of vote buying, turnout buying, and double persuasion.

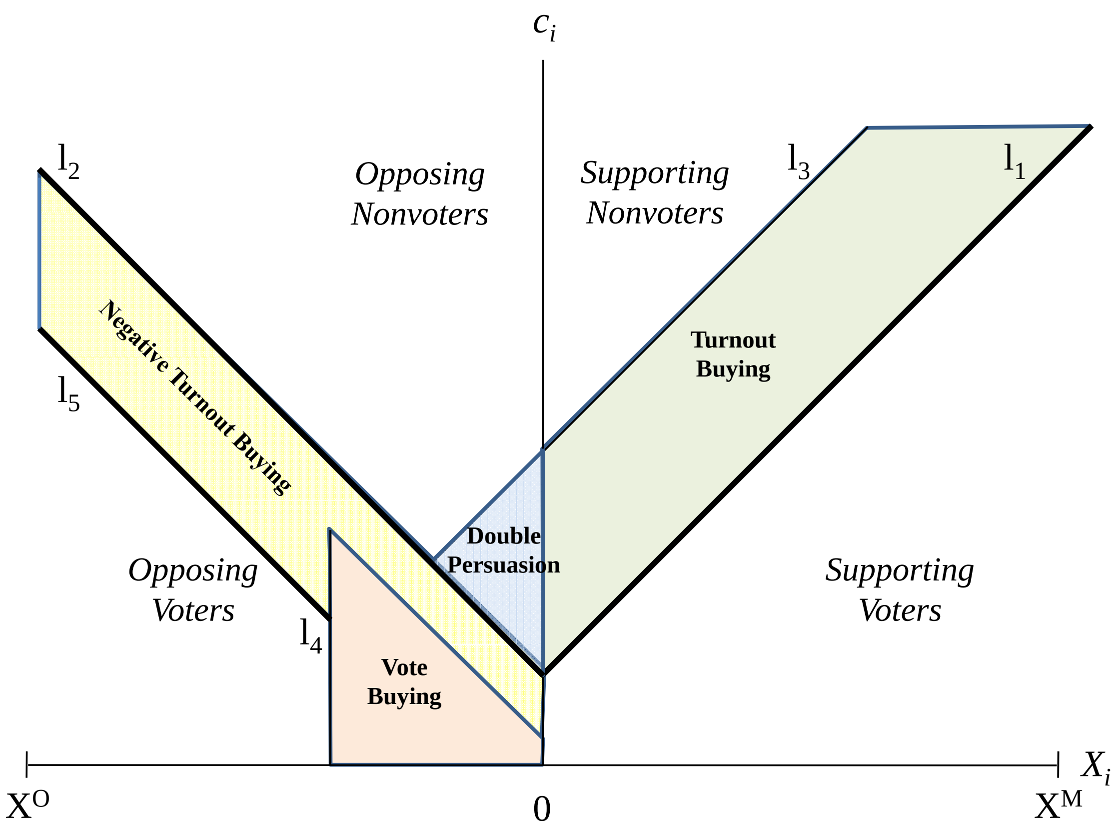
4.9 Negative Turnout Buying

Thus far, our model has incorporated three strategies available to parties during electoral campaigns—vote buying, turnout buying, and double persuasion. A revised version of this paper will also examine an additional strategy, negative turnout buying. Using this strategy, parties can reward opposing or indifferent voters for staying home on Election Day. This requires an additional assumption not employed in the above analysis: the party can write enforceable contracts with citizens providing rewards in exchange for not turning out. We provide a preliminary graphical representation of negative turnout buying in Figure 6.

Negative turnout buying produces one net vote for the machine, but unlike turnout buying and double persuasion, it does so by reducing the opposition's support, not by increasing support for the machine party. At the margin, we would thus expect the party to be willing to pay the same for the final opposing voter it demobilizes as for the final supporting nonvoter it mobilizes (i.e., turnout buying).

The model is made more complicated, however, by the fact that both negative turnout buying and vote buying target the same set of individuals—opposing voters. By incorporating negative turnout buying, the model must examine how the party allocates resources given that the party faces a triple choice with each opposing voter—(1) not rewarding her at all, (2) rewarding her for staying home on Election Day, or (3) rewarding her for switching her vote.

Figure 6: Targeting Citizen Types, with Negative Turnout Buying (Illustrative)



5 Conclusion

Although the existing literature reveals at least five distinct strategies by which parties distribute rewards during election, few scholars have examined how parties trade off amongst these strategies. The model developed above builds on Cox (2006) by explicitly incorporating both strategies of persuasion and mobilization. Our model provides insight into how parties allocate resources across three strategies—vote buying, turnout buying, and double persuasion. A forthcoming version of this paper will also incorporate the strategy of negative turnout buying.

An important direction for future research is testing the comparative statics of our model. The quantitative analysis of panel data offers an important opportunity to test whether the predictions of our model reflect how parties actually target rewards. For example, panel surveys can help to address endogeneity by capturing *ex ante* partisan preferences (i.e., opinions before receiving rewards) that indicate whether rewards target machine supporters or opponents. To date, research on this topic has been hampered by a lack of panel data on rewards given during elections. Notable exceptions are the Mexico 2000 and 2006 Panel Studies (Lawson et al. 2000; Lawson et al. 2007). Additional panel studies would provide a valuable contribution by offering comparable data across countries.

A second direction for future research would be to incorporate rewarding loyalists into our modeling framework. Although scholars have advanced scholarly research by providing potential explanations for rewarding loyalists, there remains little consensus and no studies have explored how parties might trade off between rewarding loyalists and other strategies. Our model does not include rewarding loyalists, but we fully acknowledge that this strategy exists in the real world and is an important avenue for future research.

Finally, a fruitful extension of our model would be to examine the effects of different distributions of political preferences and voting costs. The current version of our model focuses on analyzing how parties combine distinct reward targeting strategies, and how the combination of these strategies changes in response to the introduction of institutional parameters such as compulsory voting or the secret ballot. We thus assume that $f(x_i)$ and $g(c_i)$ are distributed uniformly, which facilitates the evaluation of comparative statics. Under this simplifying assumption, the number of voters targeted by each strategy is captured by the shaded areas in Figure 5. While this simplifying assumption does not have substantive impacts on the analysis of comparative statics, it may give inaccurate predictions about the prevalence of each strategy if it does not accurately reflect the true distribution of the population in a given country. In order to examine the extent to which parties utilize each strategy, it would be necessary to analyze distributions that better accord with these contexts, such as a bivariate normal distribution.

Understanding how parties choose among different reward strategies during campaigns has important policy implications. Numerous countries have taken concerted efforts to stamp out "vote buying" (Schaffer 2007), without the knowledge that this broad practice actually encompasses at least five distinct strategies. Further research on how each strategy is expected to respond to specific parameter shifts could help to inform policymakers about the potential ramifications of future interventions.

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