Chapter 13

Knowledge and the Environment: Markets with Privately Produced Public Goods

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13.1 Privately Produced Public Goods

What do environmental emissions have in common with knowledge? This chapter sees both as *privately produced public goods* [1] and gives conditions for efficient allocation of resources in economies with such goods. These conditions are independent of the units of measurement and extend those of Lindahl, Bowen, and Samuelson for standard public goods. The motivation is to understand efficiency in markets in which new types of items such as knowledge and environmental assets are traded along with standard private goods. Both are public goods in that they are not rival in consumption. However, they are privately produced and thus differ from classical public goods that are produced by governments.

Following Chichilnisky, Heal, and Starrett [1], we consider competitive markets, in which every trader faces the same price for each good. The institutional structure for trading public goods contemplated here is similar to that of the emissions markets for sulfur dioxide (SO_2) that are traded in the Chicago Board of Trade since 1993. The example of global emissions markets is especially interesting. These were created recently by Article 17 of the Kyoto Protocol, where 166 nations explicitly agreed on the creation of such tradable rights among Annex B countries, which are mostly industrial nations (see chapters 11, 12, and 14 of this volume, and the Appendix). These markets were

[1] For the foundations of economies with public goods, the reader is referred to the excellent work of Laffont [4,5] and Varian [6].

formally proposed by the scientists of Columbia's Program on Information and Resources (PIR) to the United Nations Framework Convention on Climate Change in May 1994 and emerged in December 1997 in the Kyoto Protocol (see, e.g., Chichilnisky [2]).

Another example of a privately produced public good is the total amount of knowledge in society. In idealized terms this can be represented by products (e.g., software) that can be duplicated at no cost, so the good is not rival in consumption. Knowledge is often privately produced, thus satisfying the definition of privately produced public good that is provided here. In the case of knowledge, the traders' property rights could be interpreted as rights to use a certain number of licences for knowledge products (e.g., software; see Chichilnisky [3]). In all cases the markets considered here are competitive throughout.

13.2 Markets with Privately Produced Public Goods

We use the model of chapter 3 of this book. A competitive market has $J \geq 2$ traders, one public good denoted $a \in R$, for example, the concentration of carbon dioxide (CO_2) in the atmosphere, and a private good denoted $c \in R$. The property rights of the trader j, $\overline{a}_j$, restrict the amount of carbon that the trader has the right to emit at no cost. More rights can be bought or sold in a competitive market. Each trader produces an amount of the private good $c_j^* \in R$, chosing an input of the public good a_j^* that maximizes profits. Formally,

$$a_j^* = \arg \max_{a_j \in (0,\ \infty)} [\phi_j(a_j) - \pi a_j],$$

where π is the relative price of a, $c_j = \phi_j(a_j)$ and $\partial c_j / \partial a_j < 0$. Private goods are the numeraire, that is $p_c \equiv 1$, and $(\overline{a}_j - a_j)$ represents the amount of carbon emitted by trader j to produce private goods over and above its initial rights. Each trader chooses his or her consumption of private and public goods c_j^*, $a^* \in R^{N+1}$ so as to maximize utility

$$\max_{c_j, a} u_j(c_j,\ a),$$

subject to a budget constraint:

$$c_j + \pi(\overline{a}_j - a_j) = \phi_j(c_j);$$

that is, the value of consumption of private and public goods equals the value of production.

In a competitive market equilibrium all markets clear:

$$\sum_{j=1}^{J} a_j^* = \sum_{j=1}^{J} \overline{a}_j = a^*,$$

and

$$\sum_{j=1}^{J} c_j^* = \sum_{j=1}^{J} \phi_j(a_j^*).$$

13.3 Efficiency Conditions

This section derives efficiency conditions for the allocation and provision of privately produced public goods. The classic Lindahl-Bowen-Samuelson conditions for Pareto efficiency in the supply of classic public goods do not in principle apply here because the public good a is privately produced: Here each producer has different production functions, and maximizes profits.

PROPOSITION 1 Efficiency requires that for every trader j, the sum over all traders of the marginal rates of substitution between the private and the public good should equal the corresponding rate of transformation ϕ_j'. In a competitive market, this rate of transformation must equal the relative price of the privately produced public good.

PROOF. By definition an allocation is Pareto efficient if there is no other feasible allocation that makes everyone as well off and someone strictly better off. By definition, therefore, at such an allocation each trader maximizes his or her utility given the (fixed) levels of utility of all others. Formally, for the J traders, a Pareto-efficient allocation $[c_1, \Sigma_{j=1}^{J} \varphi_j(c_j), \ldots, c_J, \Sigma_{j=1}^{J} \varphi_j(c_j)]$ solves the problem

$$\max_{c_1 \in R} + \left\{ u_1(c_1, \sum_{j=1}^{J} \varphi_j(c_j)) \right\}$$

$$\text{subject to} \quad \left\{ u_2[c_2, \sum_{j=1}^{J} \varphi_j(c_j)] \right\} = \overline{u_2}, \ \ldots, \left\{ u_J[c_J, \sum_{j=1}^{J} \varphi_j(c_j)] \right\} = \overline{u_J},$$

where $\varphi(c_j) = \phi_j^{-1}(c_j)$ and $\Sigma_{j=1}^{J} a_j = \Sigma_{j=1}^{J} \phi_j^{-1}(c_j) = a$.

To obtain an optimum, one considers the so-called Lagrangian expression $\{u_1[c_j, \Sigma_{j=1}^{J} \varphi_j(c_j)]\} + \Sigma_{j=2}^{J} \lambda_j \{u_j[c_j, \Sigma_{j=1}^{J} \varphi_j(c_j)]\}$, where the λ_j''s are so-called Lagrangian multipliers, and maximizes the expression

$$\max \sum_{j=1}^{J} \lambda_j \left\{ u_j[c_j, \sum_{j=1}^{J} \varphi_j(c_j)] \right\}, \qquad (13.1)$$

where $\Sigma_{j=1}^{J} a_j = \Sigma_{j=1}^{J} \phi_j^{-1}(c_j) = a$.

Optimizing (13.1), one obtains for each trader j

$$\lambda_j \partial u_j/\partial c_j = -\left(\sum_{j=1}^{J} \lambda_j \partial u_j/\partial a\right)\varphi'_j \tag{13.2}$$

or

$$\lambda_j = (-K)\frac{\varphi'_j}{\partial u_j/\partial c_j}, \tag{13.3}$$

where for all j, K is the same:

$$K = \left(\sum_{j=1}^{J} \lambda_j \partial u_j/\partial a\right).$$

Substituting (13.3) into (13.2), one obtains

$$\lambda_j \partial u_j/\partial c_j = -\left(\sum_{j=1}^{J} -K\varphi'_j \frac{\partial u_j/\partial a}{\partial u_j/\partial c_j}\right)\varphi'_j$$

or

$$\frac{\lambda_j \partial u_j/\partial c_j}{\varphi'_j} = (-K)\left(\sum_{j=1}^{J} \varphi'_j \frac{\partial u_j/\partial a}{\partial u_j/\partial c_j}\right).$$

Substituting φ'_j from (13.2), one obtains

$$\sum_{j=1}^{J} \varphi'_j \frac{\partial u_j/\partial a}{\partial u_j/\partial c_j} = 1 \tag{13.4}$$

or

$$\sum_{j=1}^{J} \frac{\partial u_j/\partial a}{\partial u_j/\partial c_j} = \phi'_j,$$

which is an expression generalizing the Lindahl-Bowen-Samuelson (LBS) conditions for Pareto efficiency in the allocation of *privately* produced public goods. It requires that for each trader j the sum of the marginal rates of substitution should equal the corresponding rate of transformation. Observe that in a competitive market, this equals the relative market price of the privately produced public good but not otherwise, and this condition need not be similar to the LBS condition. This completes the proof. ■

REMARK 1 Observe that expression (13.4) is independent of the units of measurement and it does not depend on the weights λ_j.

References

1. Chichilnisky, G., G. Heal, and D. Starrett. “International Markets with Emissions Trading: Equity and Efficiency.” Discussion Paper No. 81, Center for Economic Policy Research, Stanford University, fall 1993. (Chapter 3 of this volume)
2. Chichilnisky, G. “Development and Global Finance: The Case for an International Bank for Environmental Settlements.” Discussion Paper No. 10, Office of Development Studies of UNDP, UNESCO, New York, 1997. (Chapter 11 of this volume)
3. Chichilnisky, G. “Markets for Knowledge.” Invited lecture at the National Academy of Sciences Forum “Nature and Human Society: The Quest for a Sustainable World,” October 27–30, 1997. Published in its proceedings, ed. P. Raven, National Academy of Sciences, Washington, D.C., 2000.
4. Laffont, J.-J. *Effets Externes et Theorie Economique.* Monographies du Seminaire d’Econometrie. Paris: Editions du Centre National de la Recherche Scientifique, 1977.
5. Laffont, J.-J. *Fundamentals of Public Economics.* Cambridge: MIT Press, 1988.
6. Varian, H. *Microeconomic Analysis.* New York: W. W. Norton, 1984.