Chapter 6

Efficiency Properties of a Constant-Ratio Mechanism for the Distribution of Tradable Emission Permits

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6.1 Introduction

The world's public opinion has been increasingly alarmed by the dangers posed by carbon dioxide (CO_2) emissions. The current level of emissions, if not curbed, could lead to relevant climate changes that might have disastrous effects on humanity. Chichilnisky [3] and Chichilnisky and Heal [4] offer a general review of the problem of CO_2 emissions. Such a complex issue can be analyzed from several viewpoints. This chapter focuses on the public good aspect. As CO_2 tends to distribute itself evenly in the atmosphere over time, in the long run it does not matter where on the earth's surface CO_2 originates; what matters is only the global amount of emissions. Carbon dioxide closely approximates a global public good.

To curb or at least slow the growth of CO_2 emissions, a mechanism needs to be devised to deal with the public good problem. Two possibilities are direct regulation and discouraging taxation. A third possibility, which forms the object of this chapter, follows the Coasian tradition and consists of distributing tradable emission permits.

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In the simplest version a tradable emission permit mechanism would work as follows. An international market for emission permits is set up. Each country receives a given amount of emission permits. If a country pollutes more than its amount of permits allows for, it should make up for the difference by buying permits on the international market. If it pollutes less, it can sell the unused permits. It is common wisdom that such a mechanism would bring about production efficiency: Countries will face a powerful incentive to develop and apply low-pollution technologies.

However, Chichilnisky, Heal, and Starrett [5] examine the problem of Pareto efficiency for a tradable emission permit mechanism and prove that, given a global level of emissions and a distribution of tradable permits, the competitive equilibrium allocation is, in general, not Pareto efficient. To reach a Pareto-efficient allocation, the planner needs to look for some special permit allocation.

This chapter takes a different perspective on the same problem. Instead of holding the global level of emissions constant and looking for the "right" distribution, the reverse is done; that is, given some exogenous ratios, the aim is to find a global level of emissions that gives a Pareto-efficient allocation. The main proposition is that, given regularity conditions, that level exists and is unique.

A constant-ratio mechanism has three logical stages. First, each country is exogenously entitled to a constant ratio of all emission permits that will be issued. Second, the planner chooses the total amount of emissions. Third, each country receives its share of permits and is free to trade them for consumption goods. A constant-ratio mechanism can be seen as a way to separate the distribution issue from the efficiency issue.

The starting point of a constant-ratio mechanism is the definition of property rights over a special factor of production, emissions, which is similar to the definition of property rights over other factors, say, offshore oil. One possibility is to define such property rights as a set of ratios of any future emission that each country is entitled to.[1] For instance, one country could be entitled to 10% of all the world's emission permits whatever the global level of emissions will be. An entitlement to a constant ratio of emission permits has an analogy to a property right over a corresponding fraction of the atmosphere. A country entitled to 10% of all the world's permits could be viewed as the owner of 10% of the atmosphere. Obviously, this property right is incomplete,

[1] For instance, candidates for proportions could be population shares or current emission shares.

as a country cannot decide independently the level of pollution of its share of the atmosphere.

The mechanism lets countries trade their permits. A country can pollute more than its share allows for by buying permits from another country or, conversely, can pollute less than its share and sell part of its permits. Then, as is easy to see, the marginal productivity of emissions will be equalized to the international price of emission rights in all countries. Permit trade alone guarantees efficiency on the production side.

However, as Chichilnisky, Heal, and Starrett have shown in chapter 3 of this volume, if we look at the consumption side, we run into the public good problem, and we see that, in general, countries are not satisfied with the allocation that results from a given level of emissions through a competitive equilibrium. Suppose, for instance, that an overwhelming majority of countries want to decrease the current level of emissions while only a minority want to keep it constant or to increase it (of course, each country has taken into account the effect that a decrease will produce on its utility both directly as a decrease of a public bad and indirectly through a decrease in consumption due to less available production factor). Then, if side transfers are possible, there exists an alternative allocation at which the emission level is decreased and the majority of countries that benefit from the decrease compensate with consumption goods the minority that are hurt. Such an alternative allocation is Pareto improving. Therefore, to be efficient, a level of emissions needs to be resistant to recontracting among countries. In an intuitive sense the global emission level must be such that the thrust of the countries that want to increase it exactly offset the thrust of the countries that want to decrease it. In this chapter such a concept is formalized by a marginal willingness-to-pay function.

Section 6.2 contains the main propositions. It is shown that, in the constant-ratio mechanism, for each vector of ratios, there exists a unique global level of emissions that results in a Pareto-efficient allocation. Pareto efficiency is defined in the broadest sense. In the hope of making the exposition more intuitive, the proof is given for a world with N countries, one private good and one factor of production (emissions). The Appendix generalizes the result to a model with several private goods and several production factors, both traded and nontraded.

Section 6.3 touches the issue of implementation. Once it is established that a constant-ratio mechanism can reach a Pareto-efficient allocation, the question is, If countries vote on the global level of emissions, what will happen? It turns out that there exists a unique voting equilibrium at which countries vote in a straightforward manner but that this equilibrium need not coincide with the Pareto-efficient level of emissions.

6.2 Pareto Efficiency

In this deterministic[2] model there is one homogeneous consumption good, c. To produce it, it is necessary to produce polluting emissions. Emissions can be regarded as a production factor for c. Given a technology, if we want to produce more c, we need to pollute more. In this simplified model emissions will be the only argument of the pollution function. Utility depends on two arguments: the consumption of private good and the consumption of the public bad.

There are N countries,[3] each of which has a country-specific utility function $U_i(c_i, e)$—the arguments are the country's private consumption and the world's level of emissions—and a country-specific production function $f_i(e_i)$—the argument is the amount of emission used by the country to produce private goods. The production function is strictly concave. The utility function is increasing in c, decreasing in e, twice-continuously differentiable, and strictly quasi convex. Moreover, the marginal rate of substitution between consumption and pollution $U^i_e(c, e)/U^i_c(c, e)$ is assumed to be strictly decreasing in c and in e (i.e., $U^i_{-e}(c, -e)/U^i_c(c, -e)$ is strictly decreasing in $-e$ air quality and strictly increasing in c: both air quality and the consumption good are normal goods).[4]

Also,[5]

$$\lim_{e_i \to 0^+} f'_i(e_i) = \infty \quad \text{and} \quad \lim_{e_i \to \infty} f'_i(e_i) = 0 \qquad i = 1, 2, ..., N.$$

A constant-ratio mechanism for the allocation of emission permits determines each country's amount of permits ϵ_i as follows:

$$\epsilon_i = \pi_i e$$

[2]The double uncertainty in the connection between CO_2 emissions and global heating and between global heating and effects on human activity is a fundamental feature of the global warming issues and poses a series of problems in a dynamic context. This model is both static and deterministic.

[3]In principle, the constant-ratio mechanism should be based on people and firms and not on countries. People would be entitled to shares of the world's emission amount, which they would sell to firms. Firms would produce using permits bought from people. In this chapter, the word *agent* (be it a consumer or a producer) could as well replace the word *country.* However, all the current discussions focus on the role of countries. Therefore, this model will be based on countries with all the caveats that aggregate utility functions entail.

[4]This assumption is used to prove uniqueness but is not needed for existence.

[5]Assumption

$$\lim_{e_i \to \infty} f'_i(e_i) = 0$$

can be replaced with

$$\lim_{e_i \to \infty} f_i{}'(e_i) = 0 \quad \text{with a} \in (\infty, 0].$$

with

$$\pi \geq 0, \qquad i = 1, 2, ..., N \sum$$

$$\sum_{i=1}^{N} \pi_i = 1.$$

The ratios π_i are predetermined and are held constant. The countries are free to trade their share of emission permits.

In this model two factors determine a country's level of consumption: its technology and its ratio of permits. A country with an efficient technology will produce more goods. At the margin using a permit to produce goods or selling the permit for consumption good is equivalent. However, efficient countries can earn a larger surplus before they get to the margin. The second source of difference is the ratio of permits. A country with a high share of permits will either sell them for consumption good or use them to produce without the need of buying permits from other countries. In the general model, treated in the Appendix, the differences between countries will also depend on the endowments of factors of production.

Finally, the model includes a planner, whose only decision variable is the total level of emissions

$$e \in [0, \infty).$$

DEFINITION An allocation $(e; e_1, ..., e_N; c_1, ..., c_N)$ is Pareto efficient in an unrestricted sense if there does not exist a different allocation, that may involve side transfers in consumption good, that makes no country worse off and at least one country better off.

DEFINITION At a given e, a competitive equilibrium is given by

$$< c_1^*(e), ..., c_N^*(e); e_1^*(e), ..., e_N^*(e); p(e) >$$

that satisfy, for $i = 1, 2, ..., N$,

$$V_i(e) = \max Ui(ci, e) \quad \text{subject to} \quad c_i - f_i(e_i) = p(\pi_i e - e_i), \qquad e_i \geq 0$$

and

$$\sum_{i=1}^{N} e_i = e.$$

The term $V_i(e)$ is the maximized utility function for country i and depends on e and on all the π's.

Notice that, for any global level of emissions $e \in [0, \infty)$, the necessary conditions for competitive equilibrium correspond to production efficiency:

$$f_i'(e_i^*(e)) = p \quad \text{for } i = 1, 2, ..., N$$

LEMMA 1 Given e, there exists a unique competitive equilibrium ($c_1^*(e)$, ..., $c_N^*(e)$; $e_1^*(e)$, ..., $e_N^*(e)$; $p(e)$).

PROOF. When e is held constant, this model has one good (c), one factor of production (e), N producers, and N consumers (consumer i is entitled to all the profits of producer i and none of the profits of producers j i).

The assumptions that $f_i' < 0$ and that

$$\lim_{e_i \to 0^+} f_i'(e_i) = \infty \quad \text{and} \quad \lim_{e_i \to \infty} f_i'(e_i) = 0 \qquad i = 1, 2, ..., N$$

ensure that the solution to $f_i'(e_i) = p$ exists and is unique in all countries. Therefore, the solution to the equation

$$\sum_{i=1}^{N} e_i^*(p) = e$$

exists and is unique. As the $c_i^*(e)$ are uniquely determined by the trade balance constraints, it follows that a competitive equilibrium exists and is unique. ■

DEFINITION The marginal willingness-to-pay function for country i is defined as

$$MW_i(e) = \frac{V_i'(e)}{U_c^i(c_i^*(e), e)}.$$

When it is positive (negative), $MW_i(e)$ represents the amount of consumption that good country i is willing to forgo in exchange for a marginal increase (decrease) in the total emission level e.

Let us pause on the interpretation of MWi. By the envelope theorem,

$$\frac{dc_i^*(e)}{de} = \pi_i p(e).$$

Then, for country i,

$$MW_i(e) = \frac{U_e'(c_i^*(e), e)}{U_c^i(c_i^*(e), e)} + \pi_i p(e).$$

The first addend corresponds to the marginal rate of substitution between global emission level and consumption for country i. As U_e^i is negative, the first addend is negative. On the other hand, the second addend is positive and decreases as e increases. At $\hat{e}$,

$$-\frac{U_e'(c_i^*(\hat{e}),\ \hat{e})}{U_c^i(c_i^*(\hat{e}),\ \hat{e})} = \pi_i p(\hat{e}),$$

so that $MW_i(\hat{e})$. Given that vector of ratios, $\hat{e}_i$ is the bliss point for country i. If $e > \hat{e}$, then country i would like to see e decrease and vice versa. In general $\hat{e}_i$ will differ from country to country.

So far we have looked at a single country. If we turn to the aggregate, we can imagine that the efficient e will be such that the pressure from countries who want a higher e equals the pressure from countries who want a lower e. To formalize this concept we will use the notion of marginal willingness-to-pay aggregate function, defined as

$$MW(e) = \sum_{i=1}^{N} MW_i(e).$$

The term $MW(e)$ can be viewed as a general willingness to move e. For instance, if, at e, $MW(e)$, then a new allocation, possibly including side transfers, can be found at $\hat{e} > e$ such that all countries are better off.

LEMMA 2 $MW(e)$ is continuous and strictly decreasing and there exists a unique $\hat{e}$ such that $MW(\hat{e})$.

PROOF. To prove continuity, consider

$$\begin{aligned} MW(e) &= \sum_{i=1}^{N} MW_i(e) \\ &= \sum_{i=1}^{N} \frac{U_e^i(c_i^*(e),\ e)}{U_c^i(c_i^*(e),\ e)} + f_j'(e_j^*(e)) \quad \text{for any } i = 1,\ 2,\ ...,\ N. \end{aligned}$$

By assumption, U_e^i, U_c^i and f_j' are continuous, and $U_c^i > 0$. Therefore, $MW(e)$ is continuous.

To prove the "strictly decreasing" part, it will be proven that both addends are strictly decreasing. First, let us prove that the first addend is decreasing for each country. Recall that the marginal rate of substitution is decreasing in both c and e and that

$$\frac{dc_i^*(e)}{de} = \pi_i p(e).$$

Therefore,

$$\frac{d\dfrac{U_e^i}{U_c^i}}{de} = \frac{\partial\dfrac{U_e^i}{U_c^i}}{\partial e} + \frac{\partial\dfrac{U_e^i}{U_c^i}}{\partial c}\frac{dc_i^*(e)}{de} < 0 \quad \text{for } i = 1, 2, ..., N.$$

Next let us prove that the second addend is decreasing as well. Consider

$$p(e) = f_i'(e_i^*(e))$$

$$\frac{dp}{de} = f'\frac{de_i^*}{de}$$

$$\sum_{i=1}^{N}\frac{de_i^*}{de} = 1$$

$$\sum_{i=1}^{N}\frac{1}{f_i'}\frac{dp}{de} = \frac{dp}{de}\sum_{i=1}^{N}\frac{1}{f_i'} = 1.$$

Because $f_i' < 0$ for all i, then dp/de. Therfore, $MW(e)$ is (strictly) decreasing.

To prove existence and uniqueness, recall that

$$\lim_{e_i\to 0^+} f_i'(e_i) = \infty \quad \text{and} \quad \lim_{e_i\to\infty} f_i'(e_i) = 0 \qquad i = 1, 2, ..., N.$$

Furthermore, as by assumption $U_e^i(0, e)/U_c^i(0, e)$ is negative and decreasing in e, then

$$\lim_{e_i\to 0^+}\frac{U_e^i(0, 0)}{U_c^i(0, 0)}$$

is a bounded negative number. Therefore,

$$\lim_{e\to 0^+} MW(e) = +\infty$$

$$\lim_{e\to\infty} MW(e) \leq 0.$$

Then, because $MW(e)$ is continuous, there exists an $\hat{e}$ such that $MW(\hat{e}) = 0$. As $MW(e)$ is also strictly decreasing, $\hat{e}$ is unique.

The properties of the marginal willingness-to-pay function proven here provide a tool for demonstrating the main result of this chapter.

PROPOSITION 3 In a constant-ratio mechanism there exists a unique global level of emissions $\hat{e}$ that results in a competitive equilibrium corresponding to a Pareto-efficient allocation.

PROOF. To prove the proposition we state the conditions for unrestricted Pareto efficiency and show that there exists a unique level of emissions $\hat{e}$ such that a constant-ratio mechanism allocation satisfies those conditions.

By lemma 1, for a given e, a constant-ratio mechanism results in a unique competitive equilibrium allocation

$$< e_1^*(e), \ldots, e_N^*(e); c_1^*(e), \ldots, c_N^*(e), e.$$

Given the convexity of the problem, the first-order conditions for Pareto efficiency are necessary and sufficient. The conditions for unrestricted Pareto efficiency (the planner chooses all the variables) are

$$f_1'(e_1) = f_2'(e_2) = \cdots = f_N'(e_N), \tag{6.1}$$

$$\lambda_1 U_c^1(c_1, e) = \lambda_2 U_c^2(c_2, e) = \cdots = \lambda_N U_c^N(c_N, e), \tag{6.2}$$

and

$$\sum_{i=1}^{N} \lambda_i U_e^i(c_i, e) + f_j'(e_j) = 0 \quad \text{for any } j. \tag{6.3}$$

First, notice that (6.1) is always satisfied by $e_1^*(e), \ldots, e_N^*(e)$. Now consider all the possible constant-ratio mechanism allocations: $< e_1^*(e), \ldots, e_N^*(e), c_1^*(e), \ldots, c_N^*(e), e >$.

CLAIM 1 If $e = \hat{e}$ and $\lambda_i (1/U_c^i(c_i^*(\hat{e}), \hat{e}))$ for $i = 1, 2, \ldots, N$, then $< e_1^*(e), \ldots, e_N^*(e), c_1^*(e), \ldots, c_N^*(e), e >$ satisfy (6.1) to (6.3).

PROOF. Equation (6.1) is always satisfied. Obviously, (6.2) is satisfied. With these λ's, (6.3) coincides with $MW(e) = 0$, which, by lemma 2, is satisfied if $e = \hat{e}$.

CLAIM 2 If $e \neq \hat{e}$ or $\lambda \neq a\,(1/U_c^i(c_i^*(\hat{e}), \hat{e}))$ for $i = 1, 2, ..., N$ $a \in [0, \infty)$, then $< e_1^*(e), ..., e_N^*(e), c_1^*(e), ..., c_N^*(e), e >$ cannot satisfy (6.1) to (6.3).

PROOF. If $\lambda \neq a\,(1/U_c^i(c_i^*(\hat{e}), \hat{e}))$, then (6.3) does not hold, and claim 2 is proven. Suppose that $\lambda_i = a\,(1/U_c^i(c_i^*(\hat{e}), \hat{e}))$. Then (6.3) coincides with $MW(e) = 0$. However, by lemma 2, if $e \neq \hat{e}$, then $MW(e) \neq 0$, and (6.3) does not hold.

Claim 1 proves existence. Claim 2 proves uniqueness. ■

For the sake of exposition, the proof was given for a one-good, one-factor model. However, it is possible to generalize the assumptions of the model. Suppose there is a vector of consumption goods c, a vector of internationally traded factors of production k, and a vector of noninternationally traded factors of production l. Proposition 1 still holds. For the proof, see the Appendix.

6.3 Implementation

So far it has been assumed that a planner is to choose the global level of emissions. Then, given a set of ratios, this planner can always find a Pareto-efficient level. However, the planner needs to know every country's utility function and production set, which is a heavy informational requirement. Is it possible to decentralize the choice of the emission level?

In this section majority voting sets the global emission level.[6] Each country has one vote. Given a level e', another level e'' is proposed, votes are taken, and the level that receives the greater number of votes is implemented. Successive rounds of voting are taken until a global level of emission e^M is reached such that no other e can get a greater number of votes. Such a level e^M is called a voting equilibrium .

As Gibbard [6] showed, in general a unique voting equilibrium need not exist. However, a constant-ratio mechanism yields the following.

PROPOSITION 4 In a constant-ratio mechanism there exists a unique voting equilibrium e^M, where e^M is the global emission level desired by the median voter.

[6] Bowen [2] studied the problem of voting on the level of a public good to be provided through taxation. Citizens share the tax burden equally. Here the problem is analogous. A public good, clean air, is provided through taxation in predefined ratios. The only difference is that whereas in Bowen's model taxation hits a consumption good, here it hits a production factor.

PROOF. Consider the function $V_i(e)$ defined by

$$V_i(e) = \max U^i(c_i, e) \quad \text{subject to } c_i - f(e_i) = p(\pi_i e - e_i), \qquad e_i \geq 0.$$

The term $V_i(e)$ is continuous. By extending lemma 2, for each i, there exists a unique $\hat{e}$ such that $MWi(\hat{e}) = 0$, which implies that there exists a unique $\hat{e}_i$ such that $V_i^i(\hat{e}) = 0$. Then $V_i(e)$ is single peaked for all countries. Then[7] there exists a unique voting equilibrium e^M, where e^M is the global emission level desired by the median voter. ■

In general, the voting equilibrium e^M will be different from the Pareto-efficient level $\hat{e}$. The condition that determines e^M is

$$MW_M(e) = \frac{U_e^M(c_M^*(e), e)}{U_c^M(c_M^*(e), e)} + \pi_M p(e) = 0,$$

where M is the median voter and the condition that determines $\hat{e}$ is

$$MW(e) = \sum_{i=1}^{N} \frac{U_e^M(c_i^*(e), e)}{U_c^M(c_i^*(e), e)} + p(e) = 0.$$

Under some simplifying analytical assumptions, it is possible to state an intuitive condition under which majority voting yields the efficient level.

PROPOSITION 5 If all countries have identical isoelastic utility functions and receive equal ratios of emission permits, then the voting equilibrium e^M and the Pareto-efficient level $\hat{e}$ coincide if and only if the mean income and the median income coincide.

The assumption of identical utility function corresponds to assuming that differences in the way countries value clean air are due only to income differences. If two countries have the same income, they demand the same amount of clean air. This excludes cultural differences, that is, cases in which citizens of some countries might value clean air over consumption intrinsically more than citizens of other countries. Of course, technological differences are still present.

PROOF. Suppose that

$$U_i(c_i, e) = (E - e)^a c_i^b.$$

[7] As proven in Black [1].

Then

$$\frac{U_e^i(c_i^*(e),\ e)}{U_c^i(c_i^*(e),\ e)} = \frac{ac_i}{b(E - e)},$$

so that Pareto efficiency implies that

$$\frac{U_e^M(c_M^*(e),\ e)}{U_c^M(c_M^*(e),\ e)} + \pi_M p(e) = -\frac{ac_i}{b(E - e)} + \frac{1}{N}p(e) = 0,$$

whereas the voting equilibrium requires that

$$\sum_{i=1}^{N} \frac{U_e^i(c_i^*(e),\ e)}{U_c^i(c_i^*(e),\ e)} + p(e) = -\sum_{i=1}^{N} \frac{ac_i}{b(E - e)} + p(e) = 0$$

and

$$-\frac{aE(c)}{b(E - e)} + \frac{1}{N}p(e) = 0,$$

and the two conditions are identical if and only if $E(c) = c_M$. As there are no savings, the voting equilibrium e^M and the Pareto-efficient level $\hat{e}$ coincide if and only if the mean income and the median income coincide. ■

If the income distribution is skewed toward lower incomes, as the world distribution is, then the mean income is higher than the median income. Proposition 3 indicates that the global level of emission achieved through a voting equilibrium will not be Pareto efficient. Given the voting equilibrium, there could be a Pareto-improving alternative allocation whereby developed countries transfer income toward developing countries in exchange for a decrease in the global level of emissions. Therefore, a constant-ratio mechanism, if implemented through voting, is likely to bring about a global emission level that is higher than the one that an omniscient planner would choose.

6.4 Remarks and Conclusions

The result of existence of a Pareto-efficient allocation is very robust. Mainly, it depends on the fact that, if $e = 0$, all countries want e to increase, whereas if e is large enough, all countries want e to decrease. It is easy to see that existence still holds if we take the share of emission permits to be functions instead of constants.

A mechanism for dealing with public goods should have two desirable properties. First, it should separate the issue of efficiency from the issue of equity. Second, it should be implementable with decentralized information.

Regarding the first property, a constant-ratio mechanism is entirely satisfactory. The issue of equity involves selecting a vector of ratios. The fundamental question of the choice of the ratios is outside the scope of this chapter. However, once the vector of ratios is determined, the issue of efficiency can be solved uniquely and no recontracting can make countries better off.

Regarding the second property, a constant-ratio mechanism yields mixed results. On the bright side it has a unique voting equilibrium in which countries vote in a straightforward manner. However, this equilibrium need not coincide with the efficient level. The gap between the two depends on the difference between the zeroes of the marginal aggregate willingness to pay and the median willingness to pay.

Appendix

Suppose there are N countries, M consumption goods c, Q internationally traded production factors k, and P noninternationally traded production factors l. There are MN production functions, one for each country and each good.

Proposition 1 holds.

PROOF. Here the predicate of lemma 1 will be assumed, not derived; namely, it will be assumed that, for each level of e, there exists a unique competitive equilibrium allocation.[8]

Besides the respect of constraints, the conditions for a competititive equilibrium, given e, are

$$p^j \frac{\partial f_i^j}{\partial e_i^j} = p \qquad i = 1, ..., N, j = 1, ..., M, \tag{A6.1}$$

$$p^j \frac{\partial f_i^j}{\partial k_i^h} = q^h \qquad i = 1, ..., N, j = 1, ..., M, h = 1, ..., Q, \tag{A6.2}$$

$$\frac{\partial f_i^j}{\partial l_i^g} = \chi_i^g \qquad i = 1, ..., N, j = 1, ..., M, g = 1, ..., P, \tag{A6.3}$$

[8]The analysis of the conditions for existence and uniqueness of competitive equilibrium is outside the scope of this chapter. What we want to prove is that, if existence and uniqueness are already there, then a constant-ratio mechanism will preserve them.

$$\frac{\partial U^i}{\partial c_i^j} = \gamma_i p^j \qquad i = 1, ..., N, j = 1, ..., M, \tag{A6.4}$$

$$\sum_{g=1}^{P} \sum_{j=1}^{M} (\bar{l}_i^g - l_i^g) = 0, \tag{A6.5}$$

and

$$\sum_{j=1}^{M} p^j(c_i^j - f_i^j) = p\left(\pi_i e - \sum_{i=1}^{M} e_i^j\right) + \sum_{h=1}^{Q} \sum_{j=1}^{M} q_h(k_i^h - k_i^{hj}) \quad \text{for } i = 1, ..., N, \tag{A6.6}$$

where p is the price of emission permits, $(p^1, p^2, ..., p^M)$ are the price of consumption goods, and $(q^1, q^2, ..., q^Q)$ are the prices of traded factors. The γ's and χ's represent Lagrange multipliers. The first three conditions correspond to efficiency in production, the fourth condition ensures efficiency in consumption bundles, the fifth condition corresponds to the constraints for nontraded resources, and the sixth condition corresponds to the satisfaction of trade balance for each country. A competitive equilibrium determines an allocation (where c^*, e^*, k^*, and l^* are matrices),

$$< c^*(e),\ e^*(e),\ k^*(e),\ l^*(e),\ e >.$$

Let us take the price of good 1 as numeraire, that is, $p^1 = 1$. The marginal willingness-to-pay function for country i is

$$MW_i(e) = \frac{V_i'(e)}{\frac{\partial U^1}{\partial c_1^i}} = \frac{\frac{\partial U^i}{\partial e} + \gamma_i(e)p(e)\pi_i}{\frac{\partial U^1}{\partial c_1^i}} = \frac{\frac{\partial U^i}{\partial e}}{\frac{\partial U^1}{\partial c_1^i}} + p(e)\pi_i = \frac{\frac{\partial U^i}{\partial e}}{\frac{\partial U^1}{\partial c_1^i}} + \frac{\partial f_i^1}{\partial e_i^1}\pi_i.$$

The marginal willingness-to-pay aggregate function is

$$MW(e) = \sum_{i=1}^{N} \frac{\frac{\partial U^i}{\partial e}}{\frac{\partial U^1}{\partial c_1^i}} + p(e) = \sum_{i=1}^{N} \frac{\frac{\partial U^i}{\partial e}}{\frac{\partial U^1}{\partial c_1^i}} + \frac{\partial f_m^1}{\partial e_m^1} \quad \text{for any } m.$$

The term $MW(e)$ is a scalar and is analogous to the one-good case. It is easy to check that lemma 2 applies and there exists a unique $\hat{e}$ such that $MW(\hat{e}) = 0$.

Now let us replicate the proof of proposition 1. The conditions for unrestricted Pareto efficiency are

$$\text{same as (A6.1–A6.5)} \tag{A6.1$'$}$$

$$\lambda_1 \frac{\partial U^1}{\partial c_1^1} = \lambda_2 \frac{\partial U^2}{\partial c_2^1} = \cdots = \lambda_N \frac{\partial U^N}{\partial c_N^1}, \tag{A6.2$'$}$$

and

$$\sum_{i=1}^{N} \lambda_i \frac{\partial U^i}{\partial e} + \lambda_m \frac{\partial U^m}{\partial c_m^1} \frac{\partial f_n^1}{\partial e_n^1} = 0 \quad \text{for any } m \text{ and for any } n. \tag{A6.3$'$}$$

Of course, in (A6.2′) and (A6.3′) any index j could substitute 1.

If we take $\lambda_i = 1/(\partial U^1/\partial c_i^1)$ for all i, we have

$$(\text{A6.3}') = MW(e) = \sum_{i=1}^{N} \frac{\dfrac{\partial U^i}{\partial e}}{\dfrac{\partial U^1}{\partial c_i^1}} + \frac{\partial f_m^1}{\partial e_m^1} \quad \text{for any } m.$$

Then, by the fact that $MW(e)$ has a unique solution $\hat{e}$, it is straightforward to see that there exists a unique case where $< c^*(e), e^*(e), k^*(e), l^*(e), e >$ satisfy (A6.1′) to (A6.3′), that is, when

$$e = \hat{e} \quad \text{and} \quad \lambda_i = \frac{1}{\dfrac{\partial U^1}{\partial c_i^1}} \quad \text{for } i = 1, 2, ..., N.$$

Proposition 1 holds for the general case. ■

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