Chapter 5

Equilibrium and Efficiency: International Emission Permits Markets

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5.1 Introduction

Climate change poses potential serious problems for our global community. The Intergovernmental Panel on Climate Change (IPCC) [20] predicts that the global mean temperature will rise as much as 3°C above the present value before the end of the twenty-first century if the current trend of greenhouse gas emissions persists.[1] Global warming, experts believe, could cause severe detrimental economic and ecological effects, among them being decreases in agricultural productivity, more frequent storms, and alterations of ecological systems. Although uncertainties[2] still remain in terms of both scientific evidence of the greenhouse effect and the consequences of global warming, the scale, inertia, and possibly irreversible nature of climate change has caught the attention of 157 world leaders, who gathered in Rio de Janeiro in June 1992 to sign the Framework Convention on Climate Change (FCCC), which commits parties to immediate action on the issue.

The greenhouse effect is a typical public goods problem. Gases emitted mix quite uniformly over time in the atmosphere. On the other hand, unlike other

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[1] Cline [7,8] has a detailed account of the scientific basis of the greenhouse effect.

[2] For a formulation of global environmental problems in the framework of risk analysis, see Chichilnisky and Heal [3].

public goods (e.g., national defense), greenhouse gases (GHGs) are produced privately.[3] Each country's emission contributes a small part of the total emissions; therefore, a unilateral action on the part of an individual country could hardly have an impact on the whole problem. This leads to the concept of coordinated action, or joint implementation (JI), which is a framework that accommodates coordinated national environmental policies within a group of countries to achieve some specified abatement target. Numerous authors have made the point that to cut or stabilize global GHG emissions, a minimal requirement is to bring those major emitters into an agreement.

In a world of consistent disagreement, an agreement is always difficult to reach. Some countries, especially those developing countries, have already voiced their concerns. These countries argue that the accumulation of GHGs in the outer layer of the earth is the result of industrialization of today's developed countries. If any emission cut is to be made, it should be made by those countries, not by the developing countries. The developed countries, on the other hand, agree that they are mainly responsible for the accumulation of those GHGs, but they argue that their effort alone without the cooperation from the South could not solve the problem. Who should abate? Chichilnisky and Heal [1] asked this question. For an answer, we must go back to the principles of economics.

The two standard textbook approaches to public goods problem are taxes/ subsidies of the Pigovian tradition and the introduction of property rights of the Coasian tradition.[4] In practice these translate to policies such as emission targets, domestic carbon taxes, international emission taxes, and tradable emission entitlements.[5] Although emission taxes in principle could achieve emission reductions (see Hoel [19]), it is difficult to implement them in practice, especially at the international level. In contrast the Coasian approach has several advantages: The administrative costs are low, markets are easy to organize, and the environmental uncertainty and risks can be decentralized through markets. For these reasons we discuss in this chapter only the Coasian approach to global emission reductions. It is not our claim that environmental tax policies are useless, as taxes could still be an effective option for domestic environmental management.

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[3] "Knowledge" shares many of the characteristics of the public good "emission abatement" we are discussing here. From a broad perspective knowledge is also produced privately but consumed by all. Thanks to Graciela Chichilnisky for pointing this out. In a separate paper (Lin [22]), one of the authors discusses the implications of knowledge accumulation in an economy with essential resource inputs.

[4] For a review and comparison of these two approaches, see Chichilnisky and Heal [5] and Laffont and Tirole [21].

[5] For detailed discussions of each of these policy instruments, see Hoel [18] and Grubb [12].

The idea of emission trading was first proposed in a book by Dales in 1968 in the context of water pollution control. He argued that by forming a board (water control board) that sets a water quality standard in Ontario and administers a permits trading program, the water quality in the proposed area could be controlled cost effectively. Montgomery [23] provided a rigorous theoretical treatment. Issues related to the design and management of a permit market are examined thoroughly in Tietenberg [25–27], Hahn [13,14], and Hahn and Noll [15,17]. The idea of emission permits trading was not put into practice until quite recently. The United States implemented an emission trading program for sulfur oxides[6] in 1990 after experimenting with a series of semi-cost-effective measures, such as bubbles, netting, offsets, and emission banking.[7] Some of these concepts, offsets, and emission banking, for example, are close to but different from what we call emission permit markets.

Emission permits trading at an international level poses some new questions for economists. The divorce between equity and efficiency is a central feature of classical welfare economics: In the usual competitive model, any distribution of endowments will lead to an efficient allocation of resources.[8] Recently, Chichilnisky, Heal, and Starrett[9] [6] (CHS) showed that this is not true for economies in which the provision of a privately produced public good is controlled by the trading of production quotas. They showed that the way in which a given total number of tradable permits is distributed among polluters will affect the efficiency of the market solution. It had previously been believed that efficiency and distribution are independent in permit markets. This is sometimes referred to as the Coase theorem [9] about the irrelevance of the allocation of property rights to the attainment of efficiency. Using the same CHS model, Prat [24] showed that given a fixed distribution of the initial permits, there can exist only one total emission level that permit markets can obtain.

This chapter refines the findings of CHS. We study in detail the implications for market efficiency of the selection of the initial abatement targets and/or its

[6]The trading program is required by the U.S. Clean Air Act Amendments of 1990 (Public Law 101–549), Title IV (Acid Rain Provisions), in an effort to reduce sulfur dioxide emissions by 10 million tons per year, relative to the base year 1980, by the year 2001.

[7]A bubble is a situation in which the owners of the same plant may increase the pollution at one source while making a corresponding reduction at another. Netting, a similar concept to a bubble, is the process by which a remodeled or expanded plant can escape the lengthy reviewing process by the U.S. Environmental Protection Agency (EPA) if the new plant does not increase emissions signicantly. Offsets are sale/purchase of emission quotas between companies. An emissions bank is a credit agency arranging the transaction of emission quotas.

[8]This is not true for second-best situations, in which equity and efficiency are inextricably linked.

[9]Chichilnisky, Heal, and Starrett build on a model introduced in Chichilnisky [1] and developed in Chichilnisky and Heal [2].

initial distribution among countries. In a departure from CHS, we further allow for strategic behavior of abatement participants. This behavioral assumption, as we show here, significantly restricts the role of a policymaking body, even if international income transfers are allowed, in terms of the selection of both an abatement target and the initial allocation of emission permits. In general the Nash equilibrium allocation of resources achieves Pareto efficiency only for a finite number of initial abatement targets and for each target only for a finite number of specific distributions among an infinite number of possibilities.

This chapter is organized as follows. We study the two-good (one private good, one public good) version of the CHS model in sections 5.2 to 5.7. In section 5.2 we give a brief description of our model and derive the Pareto-optimality conditions. After that we study the competitive permits market equilibrium and look for conditions in terms of the distribution of initial emission permits as necessary for market efficiency. We do the same in section 5.4, but in a Nash setting. In section 5.5 we study in more detail the efficient initial permit allocation and countries' equilibrium trade positions. In section 5.6 we extend our results derived in sections 5.4 and 5.5 by allowing for international income transfers. In section 5.7 we briefly comment on permits market efficiency in case of individual permits pricing (Lindahl equilibrium). Extension to M private goods is done in section 5.8. Most of our results survive as more private goods are added. Section 5.9 concludes.

5.2 Pareto-Efficient Allocation of Resources

Our framework originated in Chichilnisky [1] and was developed in Chichilnisky and Heal [4]. A fuller version is in CHS. For illustration we work first with the one-private-good version of the model and then, in section 5.9, extend the model to M private goods. The world economy consists of I ($I \geq 2$) countries, each endowed with one private consumption good $\bar{Y}_i$ and an abatement technology

$$a_i = \Phi_i(\bar{Y}_i - y_i), \tag{5.1}$$

which transforms the private good into abatement. The term a_i in (5.1) is country i's abatement of carbon dioxide (CO_2) emissions, and y_i is the private good available for consumption. We assume that $\Phi_i(0) = 0$, so all initial private endowment $\bar{Y}_i$ is available for consumption; that is, $y_i = \bar{Y}_i$ if there is no abatement. Some of this private good $\bar{Y}_i$ may be given up to provide a better atmosphere. In addition each country has a utility function $u_i(c_i, a)$, where $a =$

$\Sigma_i a_i$ is the total abatement and c_i is the private goods actually consumed. Finally, we assume that both $u_i(c_i, a)$ and $\Phi_i(.)$ are twice continuously differentiable, strictly concave, and increasing.

DEFINITION 1 Let $\pi_i = a_i/a$ be the actual share of emission abatement contributed by each country i. An allocation $(\{\hat{c}_i\}_{i=1,\dots,I}, \{\hat{y}_i\}_{i=1,\dots,I}, \{\hat{\pi}_i\}_{i=1,\dots,I}, \hat{a})$ is Pareto efficient if it is feasible and cannot be improved on weakly for all countries and strictly for some country i.

Feasibility in our case requires

$$\sum_{i=1}^{I} \hat{\pi}_i = 1 \tag{5.2}$$

and

$$\sum_{i=1}^{I} \hat{y}_i = \sum_{i=1}^{I} \hat{c}_i. \tag{5.3}$$

Note that the feasibility constraint (5.3) here permits unrestricted transfers of consumption between countries. Pareto-efficient allocations are the set of all solutions that maximizes the weighted welfare

$$W(c_1, \dots, c_I, a) = \sum_{i=1}^{I} \lambda_i u_i(c_i, a) \tag{5.4}$$

of I countries subject to equations (5.2) and (5.3). As usual we consider all possible welfare weights $\lambda_i \geq 0$ subject to $\Sigma_{i=1}^{I} \lambda_i = 1$. Nonnegativity constraints are understood wherever appropriate.

The following notations will simplify our presentation:

$$MRS_i(a, c_i) = \frac{\partial u_i/\partial a}{\partial u_i/\partial c_i},$$

$$MC_i(a_i) = -\frac{1}{\partial \Phi_i/\partial y_i}.$$

The second line simply says that the marginal cost, in terms of the private good, of producing a_i is just the inverse of the marginal productivity of that good at the abatement level a_i.

Finally, we need a formula to transform the measure of emission abatement into permits. For each country the two measures are clearly negatively related.

We write

$$e_i = e_i^0 - a_i, \tag{5.5}$$

where e_i^0 (a constant) is interpreted as the natural emission level, or the level at which no preventive measures are taken for pollution control, $y_i = \bar{Y}_i$. With expression (5.5), we therefore may use the terms *emissions* and *abatement level* interchangeably.

LEMMA 1 The marginal cost of emission abatement for each country at an efficient allocation equals the sum of the marginal rates of substitution (between emission abatement and the private good) of all I countries [1,4].

PROOF. The lemma implies that marginal costs are equalized across countries at a Pareto-efficient allocation. This follows from the possibility of unrestricted transfers between countries (see Chichilnisky and Heal [4] and chapter 7). Lemma 1 is derived directly from the first-order conditions to the maximization of problem 5.4. It is a simple exercise to verify that

$$MC_i(a_i) = MC_j(a_j), \qquad \forall\, i \neq j \tag{5.6}$$

and

$$\sum_{i=1}^{I} MRS_i(a, c_i) = MC_j(a_j) \quad \text{for any } j \tag{5.7}$$

are necessary conditions for Pareto optimality. ■

Equation (5.7) is the well-known Lindahl-Bowen-Samuelson condition in the public goods literature. It is really another version of marginal benefits equaling marginal cost. It says that the cost of producing one more unit of public good a by one country (it does not matter which one as long as [5.6] is satisfied) must equal the summation of marginal benefits received by all who consume it. For obvious reasons, equation (5.6) is sometimes called the production efficiency condition, whereas equation (5.7) is called the allocation efficiency condition. A Pareto-efficient allocation has the following property.

LEMMA 2 Let $(\{\hat{c}_i\}_{i=1,\ldots,I}, \{\hat{y}_i\}_{i=1,\ldots,I}, \{\hat{\pi}_i\}_{i=1,\ldots,I}, \hat{a})$ be a Pareto-efficient allocation. If there exists another allocation $(\{c_i'\}_{i=1,\ldots I}, \{y_i'\}_{i=1,\ldots,I}, \{\pi_i'\}_{i=1,\ldots,I}, \hat{a})$ that is also Pareto efficient at the same total abatement level $\hat{a}$, then $\hat{\pi} = \pi_i'$, $\hat{y}_i = y_i'$ for all i.

PROOF. First note that if $\hat{\pi} = \pi'_i$ for all i, then $\hat{y}_i = y'_i$ for all i because Φ_i maps from $\Re$ to $\Re$ and Φ_i is monotonic. We prove the lemma by contradiction. Suppose not. Because $\Sigma_{i=1}^{I} \hat{\pi}_i = \Sigma_{i=1}^{I} \pi'_i = 1$, there must exist at least one j and one k such that $\hat{\pi}_j > \pi'_j$ and $\hat{\pi}_k < \pi'_k$, or equivalently, $a_j^* > a'_j$ and $\hat{a}_k < a'_k$. Then, by concavity of functions $\Phi_i(.)$, we have

$$MC_j(\hat{a}_j) > MC_j(a'_j) \quad \text{and} \quad MC_k(\hat{a}_k) < MC_k(a'_k),$$

which contradicts the necessary condition for Pareto efficiency, $MC_j(a'_j) = MC_k(a'_k)$. This lemma establishes a one-to-one relationship between the total abatement $\hat{a}$ and the actual shares of the total abatement by individual countries $\{\hat{\pi}_i\}$. ■

The lemma further implies that the relationship between $\hat{a}$ and private production levels $\{\hat{y}_i\}$ at the Pareto-efficient allocation is also uniquely determined. Therefore, a Pareto-efficient abatement level uniquely determines the production side of the economy. Note that this result relies only on the production efficiency condition. It has nothing to do with the allocation efficiency condition, which we discuss later. Also note that if there is more than one private good, lemma 2 becomes invalid: neither $\{\hat{\pi}_i\}$ nor $\{\hat{y}_i\}$ can be uniquely determined from $\hat{a}$. This complication is discussed in section 5.8.

In the next two sections, we study emission permit markets. We first consider competitive markets in which countries take the initial abatement target as given.

5.3 Competitive Emission Permits Market

The concept of international emission permits trading is simple. A global emission level, $\bar{e}$, is chosen, and a total of $\bar{e}$ permits is issued and distributed (according to some agreed formula) among the I participating countries. A country holding an initial permit allocation $\bar{e}_i$ (allowances) and emitting e_i may sell the excess $(\bar{e}_i - e_i)$, if $\bar{e}_i$ exceeds e_i, at the price p_e in exchange for private consumption goods, or may buy $(e_i - \bar{e}_i)$ permits by selling consumption goods should the country need more. The price of the private good is taken to be one.

Country i faces a trade-off between emission and private consumption,

$$c_i - y_i = (\bar{e}_i - e_i)p_e$$

or, equivalently, in terms of abatement by using the measure transformation (5.5),

$$c_i - y_i = (a_i - \overline{a}_i)p_e. \tag{5.8}$$

Each country maximizes its utility

$$\max\ u_i(c_i, a) \tag{5.9}$$

subject to budget constraint (5.8) and production feasibility constraint $a_i = \Phi_i(\bar{Y}_i - y_i)$. Each country assumes that the aggregate emissions target is met. This is a key part of the definition of a competitive equilibrium. At the equilibrium we need, of course, equality of the aggregate production and consumption of private goods, that is,

$$\sum_{i=1}^{I} c_i = \sum_{i=1}^{I} y_i. \tag{5.10}$$

Walras's law ensures that the permits market clears as well.

Let $\theta_i = \overline{a}_i/\overline{a}$ be the initial share of abatement assigned to country i. We can rewrite the budget constraint (5.8) as

$$c_i - y_i = (\pi_i\overline{a} - \theta_i\overline{a})p_e = (\pi_i - \theta_i)\overline{a}p_e. \tag{5.11}$$

Notice embedded in this equation the private-good market clearance condition. This is evident, as $\Sigma_i(c_i - y_i) = [\Sigma_i(\pi_i - \theta_i)]\overline{a}p_e = 0$.

DEFINITION 2 Given an abatement target $\overline{a}$ and an initial distribution of the abatement, $\{\theta_i\}_{i=1,\ldots,I}$, $0 \leq \theta_i \leq 1$, $\Sigma\,\theta_i = 1$. An allocation and a price $(\{c_i^*\}_{i=1,\ldots,I}, \{y_i^*\}_{i=1,\ldots,I}, \{\pi_i^*\}_{i=1,\ldots,I}, p_e^*)$ is a competitive equilibrium if for each country i $(c_i^*, y_i^*, \pi_i^*, p_i^*)$ solves problem (5.9).

If a is taken by all countries as constant, as is in the case of CHS, then the first-order conditions to problem (5.9) are simply

$$MC_i(a_i) = p_e, \qquad \forall\, i, \tag{5.12}$$

a well-known condition.

The following lemma about the existence of a competitive equilibrium is easily established.[10]

[10] A proof to the lemma is also shown in Prat [24].

LEMMA 3 Given $\overline{a}$ and $\{\theta_i\}_{i=1,\ldots,I}$, there exists a unique competitive equilibrium.

PROOF. We show the existence and uniqueness by actually solving for the equilibrium. Because $\overline{a}$ is fixed and from (5.12) $MC_i(a_i) = p_e$ for $\forall i$, the actual abatement shares $\{\pi_i^*(\overline{a})\}_{i=1,\ldots,I}$ can be uniquely determined according to lemma 2. Price $p_e^*(\overline{a})$ then is also determined by (5.12) because $a_i^* = \pi_i^*(\overline{a}) \cdot \overline{a}$ is known. From $a_i = \Phi_i(\bar{Y}_i - y_i)$, $y_i^*(\overline{a})$ in turn is calculated. Finally, from (5.11) we have $c_i^* = y_i^* + (\pi_i^* - \theta_i)\overline{a}p_e^*$. Equilibrium $[c_i^*(\{\theta_i\}, \overline{a}), y_i^*(\overline{a}), \pi_i^*(\overline{a}), p_e^*(\overline{a})]$ thus solved is unique. ■

The next two propositions reveal how the selection of an abatement target and its initial distribution affect the efficiency property of a competitive equilibrium. In proposition 1, $\overline{a}$ is assumed to be fixed and θ_i is allowed to change. In proposition 2, the opposite is true.

PROPOSITION 1 Assume that $\overline{a}$ is given. A distribution of initial abatement $\{\theta_i\}_{i=1,\ldots,I}$ associated with $\overline{a}$ leads to a Pareto-efficient allocation of resources if and only if the equilibrium $[c_i^*(\{\theta_i\}, \overline{a}), y_i^*(\overline{a}), \pi_i^*(\overline{a}), p_e^*(\overline{a})]$ satisfies condition (5.7), or

$$\sum_{i=1}^{I} MRS_i(\overline{a},\ c_i^*) = MC_j(a_j^*) \quad \text{for some } j.$$

PROOF. A permit market equilibrium allocation is Pareto efficient if both conditions (5.6) and (5.7) are met at the equilibrium. The marginal cost equalization condition (5.6) clearly is implied by the permits market equilibrium condition (5.12). However, the allocation efficiency condition (5.7) is not guaranteed. To achieve efficiency the equilibrium associated with an initial allocation rule $\{\theta_i\}$ must satisfy (5.7). ■

In the absence of a public good the distribution of initial property rights does not matter in terms of efficiency. Any distribution $\{\theta_i\}$, where $\Sigma_{i=1}^{I} \theta_i = 1$, would lead to a Pareto-efficient allocation of resources. This is the essence of the Coase theorem. What proposition 1 says is that this is not true if a public good is privately produced. In fact an extra condition (5.7), other than the physical constraint $\Sigma_{i=1}^{I} \theta_i = 1$, must be imposed on the distribution $\{\theta_i\}$. The selection space for θ_i is reduced by one dimension because of this constraint.

COROLLARY 1 Assume that $\overline{a}$ is given. If there are only two countries, then generically only one distribution, $\{\theta_1, \theta_2\}$, of the initial abatement target $\overline{a}$ leads to a Pareto-efficient allocation of resources.

PROOF. Under the two-country assumption, two equations, constraint (5.7) and the condition $\theta_1 + \theta_2 = 1$, generally would lead to a unique solution of $\{\theta_1, \theta_2\}$. ■

We further claim that if the two countries have the same production and utility functions, then equation (5.7) must be identical to $\theta_1 + \theta_2 = 1$. In this case any distribution of initial permits will lead to Pareto efficiency.

PROPOSITION 2 Assume that $\{\theta_i\}_{i=1,\ldots,I}$ is given. There exists a unique $\overline{a}$, such that the competitive equilibrium $[c_i^*(\{\theta_i\}, \overline{a}), y_i^*(\overline{a}), \pi_i^*(\overline{a}), p_e^*(\overline{a})]$ associated with initial $\overline{a}$ and $\{\theta_i\}_{i=1,\ldots,I}$ is Pareto efficient [24].

PROOF. A detailed proof is given in Prat [24]. Here we give a sketch of the proof.

As stated in proposition 1, a permits market equilibrium achieves Pareto efficiency if

$$\sum_{i=1}^{I} MRS_i[c_i^*(\{\theta_i\}, \overline{a}), \overline{a}] = MC_j(a_j^*) \quad \text{for some } j.$$

This equation contains only one unknown, $\overline{a}$. The proof is complete if we show that the left-hand side of the equation is a decreasing function of $\overline{a}$, as we already know $MC_j(a_j^*)$ increases with $\overline{a}$. ■

5.4 Nash Equilibrium

In this section we depart from CHS by assuming that each country maximizes its utility, taking the abatements of all other countries as given. This means that in solving (5.9), each country i takes a_j $(\forall j \neq i)$ as given, or

$$\max\ u_i(c_i, a_i + \Sigma_{j\neq i} a_j) \quad \text{subject to} \quad c_i - y_i = (a_i - \overline{a}_i)p_e.$$

The necessary condition for this is

$$MC_i(a_i) = p_e + MRS_i, \qquad \forall\, i, \tag{5.13}$$

which means that, at the equilibrium, the market price for emission permits will always be higher than even the lowest marginal cost of abatement. It is also clear from equation (5.13) that marginal cost equalization—the condition required for Pareto efficiency—is not required for a Nash equilibrium. Therefore, the equilibrium allocation of resources might not be Pareto efficient. Naturally, we would ask, Could we select an abatement target and a distribution formula for the number of permits corresponding to this target such that the equilibrium resource allocation coincides with a Pareto-efficient allocation? This question indeed points directly to the most sensitive issue of any potential international abatement agreement: equity. It also suggests that we could not separate equity from efficiency.

We answer this question by taking the following approach. Pick an arbitrary total abatement level $\overline{a}$ and distribute it arbitrarily among all the member countries. Solve problem (5.9) for equilibrium abatement levels. Check to see whether the equilibrium resource allocation is Pareto efficient. Repeat the process. We are interested in the set of all possible distributions of all possible totals of emission permits that lead to Pareto-efficient outcomes. It turns out that such a set contains only one point: For only one specific total abatement level and one way of distributing it among the I countries could the permits market lead to efficiency.

Pareto efficiency requires both production and allocation to be efficient. Marginal cost equalization across countries (condition [5.6]) ensures production efficiency, and marginal benefits equaling marginal cost (condition [5.7]) ensures allocation efficiency.

We now turn to the allocation efficiency condition (equation [5.7]). As will be shown in the following, for a permit market Nash equilibrium to be compatible with this condition, we must have $MRS_i = MRS_j$, which refines the set of efficient abatement allocations and reduces the choices of aggregate abatement levels. The next lemma shows that the refined set contains at most one point.

LEMMA 4 Among all possible Pareto-efficient allocations, only one allocation $(\{c_i^*\}_{i=1,\ldots,I}, \{y_i^*\}_{i=1,\ldots,I}, \{\pi_i^*\}_{i=1,\ldots,I}, a^*)$ satisfies the condition $MRS_i = MRS_j, \forall i, j$.

PROOF. Consider a Pareto-efficient allocation at which $\forall\, i, j, MRS_i = MRS_j$.

By efficiency and lemma 1, $\Sigma_i MRS_i = MC_i \,\forall\, i$. Now consider an alternative efficient allocation at which the aggregate emission level is greater. Assume contrary to the lemma that once again $\forall\, i, j, MRS_i = MRS_j$. For all countries the abatement level will be greater (because marginal costs are equal, so that all abatement levels move together) and the production of the consumption good lower, and therefore by the concavity assumptions the marginal costs MC_i will be greater for all i. By the assumption that $\forall\, i, j, MRS_i = MRS_j$, the greater marginal costs imply that $\forall i, MRS_i$ is greater. However, for each country abatement is greater and in aggregate consumption lower. This implies that for at least one country i, abatement has risen and consumption has fallen, so that MRS_i has fallen, a contradiction. ■

PROPOSITION 3 Only at a unique total abatement level, and with a unique way of distributing it among the countries as their initial endowments, could the permits market equilibrium lead to Pareto efficiency.

PROOF. Let $(\{c_i^*\}_{i=1,\dots,I}, \{y_i^*\}_{i=1,\dots,I}, \{\pi_i^*\}_{i=1,\dots,I}, a^*)$ be the unique resource allocation under the condition of lemma 4. We prove proposition 3 by construction in two steps.

STEP 1 We show that if a permits market equilibrium exists and its allocation is Pareto efficient, then the resource allocation at the equilibrium must be $(\{c_i^*\}_{i=1,\dots,I}, \{y_i^*\}_{i=1,\dots,I}, \{\pi_i^*\}_{i=1,\dots,I}, a^*)$. Recall the Pareto-efficiency condition

$$MC_i(a_i) = MC_j(a_j), \qquad \forall\, i, j$$

and the permits market equilibrium condition

$$MRS_i(c_i, a) = MC_i(a_i) - p_e, \qquad \forall\, i.$$

An equilibrium, if it exists and is Pareto efficient, must meet both of these conditions, which would require that

$$MRS_i = MRS_j, \qquad \forall\, i, j.$$

However, this is exactly the condition required by lemma 4 leading to the unique resource allocation $(\{c_i^*\}_{i=1,\dots,I}, \{y_i^*\}_{i=1,\dots,I}, \{\pi_i^*\}_{i=1,\dots,I}, a^*)$.

STEP 2 We construct, using the permit markets equilibrium conditions, a unique price and a unique allocation of initial permits. From equation (5.13) and the budget balance condition (5.11), we easily have

$$p_e^* = MC_i(\pi_i^* \mathrm{a}^*) - MRS_i(c_i^*, a^*),$$

$$\theta_i^* = \pi_i^* - \frac{(c_i^* - y_i^*)}{p_e^* a^*}. \tag{5.14}$$

Therefore, we conclude $(\{c_i^*\}_{i=1,\ldots,I}, \{y_i^*\}_{i=1,\ldots,I}, \{\pi_i^*\}_{i=1,\ldots,I}, p_e^*)$ as the only equilibrium candidate and indeed the only Pareto-efficient equilibrium if the nonnegativeness restriction on the initial permits allocation is relaxed. This completes the proof for proposition 3. ■

A negative initial assignment of abatement target, θ_i^*, to country i would mean a credit to that country in the sense that the country has not only no obligation for its share of abatement but also the rights to claim for cash income with the mere participation in the agreement.

5.5 Characterization of Efficient Permit Markets

What we have developed so far is purely an efficiency argument. It turned out, surprisingly, that there is not much choice—no choice to be precise—for a social planner in terms of choosing either the social optimal emission target or the formula for distributing the initial permits after a target has been chosen. Both the social optimal emission level and the formula for initial permits distribution are uniquely determined following the procedures outlined in the previous section. The rest is left to the markets to decide. Under the behavior assumptions we have made about the agents, the markets should come to an efficient outcome.

We now look into the efficient permit markets arrangement in more detail. As we have said earlier, equity issue is crucial to the success of the permit markets proposed so far. More specifically, each participant of the abatement agreement will be interested in knowing who gets what share of the total abatement assignment. We postpone the direct answer to this question. Instead we ask, What will be the market position of each member of the abatement agreement at the efficient equilibrium outcome? Who are the buyers? The sellers? First we need a few definitions.

DEFINITION 3 Suppose that i and j have the same initial private endowment. Country i is said to have a more *efficient* abatement technology than j if $MC_i(y_i) < MC_j(y_i), \forall\, y_i$.

DEFINITION 4 Let $\{c_i^*\}_{i=1,\ldots,I}$ be the Pareto-efficient consumption allocation at a^* such that $MRS_i = MRS_j$. Country i is said to be more *environment conscious* than j if $c_i^* > c_j^*$.

DEFINITION 5 The unique total abatement level (emissions) as stated in proposition 3 is called *efficient abatement level (emissions);* the unique initial distribution of the efficient abatement (emissions) as stated in proposition 3 is called *efficient initial abatement distribution (permits distribution).*

Definition 3 says that an abatement technology of one country is better than another if the marginal cost of abatement is smaller for every level of private production. This is clearly a global definition. Also notice that this definition is based on the assumption that the two countries have the same initial endowment $\bar{Y}$. To compare two abatement technologies with different initial private endowments, we break the difference into two components. One is the efficiency component, as is indicated in definition 3; the other is the income component, or income effect. A change in the initial private endowment would shift the trade-off curve along the y-axis in the *a*-*y* space. Marginal cost at a given abatement level for given abatement technology will not change as the initial private endowment changes. Clearly, this definition of abatement technology efficiency does not rank all possible abatement technologies. Our results presented therefore will be indicative rather than comprehensive.

Definition 4, in contrast, is a pointwise one. It says simply the country that values more the same unit of the public good is the one that cares more about the environment. It is recognized in the definition that the consumption allocation at the *efficient abatement level* a^* is unique under the assumption of equalization of marginal rates of substitution (lemma 4).

The last definition is self-evident.

PROPOSITION 4 At an equilibrium with an *efficient initial permits distribution,*

1. of two countries with the same initial endowment in private goods and the same abatement technology, the country that is more *environment conscious* is a relative permits seller;
2. of two countries with the same abatement technology and the same *environment consciousness,* the country with less initial private endowment is a relative permits seller; and
3. of two countries with the same initial endowment in private goods and the same *environment consciousness,* the country that is more *efficient* in its abatement technology is a relative permits seller.

PROOF. Substitute $e_i = e_i^0 - a_i$ into equation (5.14) and rewrite it as

$$\bar{e}_i^* - e_i^* = -(\theta_i^* - \pi_i^*)a^* = \frac{(c_i^* - y_i^*)}{p_e^*}.$$

Country i exports emission permits if the right-hand side of the equation is positive. Country i, in a relative sense, will export more permits than j if

$$(\bar{e}_i^* - e_i^*) - (\bar{e}_j^* - e_j^*) = \frac{(c_i^* - y_i^*) - (c_j^* - y_j^*)}{p_e^*} > 0.$$

Proposition 4 is a restatement of this equation in terms of each country's relative initial private endowment, environment consciousness, and efficiency in abatement technology.

We make one observation before checking through each item of the proposition. If the abatement technology of country i is more efficient than country j, assuming that they have the same initial private endowment, then at the efficient permits market equilibrium, $y_i^* < y_j^*$. A simple argument shows why. Remember that an efficient permits market equilibrium requires that $MC_i = MC_j$. The abatement technology efficiency definition says that at every y, $MC_i < MC_j$. Because marginal cost is a decreasing function of y, $MC_i = MC_j$ holds only if $y_i^* < y_j^*$.

1. Because the two countries have the same endowment and abatement technology, their actual abatement level and therefore outputs y_i^* at the permits market equilibrium must be the same. By referring to the last equation, we know that the actual consumption level of each country will determine who, in a relative sense, exports more permits. The country that is more environment conscious, a higher c^*, will have more excess of permits.

2. Again the same environment consciousness implies that the two countries have the same consumption, $c_i^* = c_j^*$. The same abatement technology would confirm that the actual abatement levels of the two countries are also the same. Therefore, the country that has less initial private endowment will be the one that has less output of private good y^* at the equilibrium, and this country will export more permits.

3. The same environment consciousness implies that the two countries have the same consumption, $c_i^* = c_j^*$. Because the initial endowments of private goods are also the same, the observation at the beginning of the proof says that the country that is equipped with a better abatement technology has a lower output y^* and therefore more excess of permits.

This completes the proof of proposition 4. ■

Next we move to answer the question posed at the beginning of this section: Who is to get what share of the total initial permits? Or, equivalently, who is to be assigned to what share of the total initial abatement?

The difference in abatement assignment two countries *i* and *j* would receive, according to equation (5.14), is

$$\theta_i^* - \theta_j^* = (\pi_i^* - \pi_j^*) - \frac{(c_i^* - y_i^*) - (c_j^* - y_j^*)}{p_e^* a^*}. \qquad (5.15)$$

The next proposition summarizes the distribution of initial abatement assignment as is determined by each country's abatement technology, environment consciousness, and initial endowment of private goods.

PROPOSITION 5 At the market equilibrium with an *efficient initial permits distribution,*

1. of two countries with the same initial endowment in private goods and the same abatement technology, the country that is more *environment conscious* will receive less abatement assignment;
2. of two countries with the same abatement technology and the same *environment consciousness,* the country with less initial private endowment will receive less abatement assignment; and
3. of two countries with the same initial endowment in private goods and the same *environment consciousness,* the country that is less *efficient* in its abatement technology will receive less abatement assignment, assuming that the number of participating members in the abatement agreement is sufficiently large.

PROOF.

1. Because the two countries have the same abatement technology, by definition 1, $\pi_i^* = \pi_j^*$. Further, we have $y_i^* = y_j^*$ because of the equality of their endowments $\bar{Y}_i$ and $\bar{Y}_j$. Equation (5.15) is then reduced to $\theta_i^* - \theta_j^* = -[(c_i^* - c_j^*)/p_e^* a^*]$, which means that the country that is more environment conscious (i.e., higher c_i^*) will be assigned less abatement.
2. The same environment consciousness and same abatement technology, under our definitions, means that $c_i^* = c_j^*$ and $a_i^* = a_j^*$. Further, $\bar{Y}_i < \bar{Y}_j$ implies that $y_i^* < y_j^*$ because $MC_i = MC_j$. It is then evident that the country that has less initial private endowment will have less private good *y* and receive less abatement assignment.
3. The same environment consciousness, by definition 2, implies that $c_i^* = c_j^*$. The difference in initial abatement assignments is then reduced to

$$\overline{a}_i^* - \overline{a}_j^* = (a_i^* - a_j^*) + \frac{(y_i^* - y_j^*)}{p_e^*} = \frac{(p_e^* a_i^* + y_i^*) - (p_e^* a_j^* + y_j^*)}{p_e^*}.$$

Substituting p_e^* by $(I - 1/I)\,MC^*$ and rewriting the last part of the previous equality, we have

$$\overline{a}_i^* - \overline{a}_j^* = \frac{\frac{I-1}{I} MC^*(a_i^* - a_j^*) - (y_j^* - y_i^*)}{p_e^*}.$$

Let i be the country with a more efficient abatement technology, and let $\hat{a}_i = \Phi_i(Y - y_j^*)$. By definition 1, $\hat{a}_i > a_j^*$. We need to prove that $\overline{a}_i^* - \overline{a}_j^* > 0$, which will be true when I is sufficiently large if we can show that $MC^*(a_i^* - a_j^*) - (y_j^* - y_i^*) > 0$ or that $(MC^* a_i^* + y_i^*) - (MC^* a_j^* + y_j^*) > 0$. The last inequality is indeed true because

$$MC^* a_i^* + y_i^* > MC^* \hat{a}_i^* + y_j^* > MC^* a_j^* + y_j^*.$$

This completes the proof of proposition 5. ■

Proposition 5 provides some insights about efficient initial abatement assignments. A preliminary judgment seems to suggest that the allocation formula of initial abatement will not be biased against either developed or developing countries. A typical less developed country is characterized by less environment consciousness and a less efficient abatement technology, two factors that cancel each other out in terms of their roles in determining the initial abatement assignments. The size of initial endowment of private goods, an attribute that could go to either developed countries or developing countries, although more likely to the latter, is positively correlated to the initial abatement assignment.

Take the example of China and the United States. The former would receive more abatement assignment on the basis of its *environment consciousness.* On the other hand, because China has less initial private endowment and is less *efficient* in its abatement technology, our proposition would suggest that more initial abatement be assigned to China. The net result is not clear without real numbers plugged in to the efficient allocation formula.

5.6 Emission Permits and/or Income Transfers

So far we have limited the policy instrument to the distribution of initial permits only. We add one more instrument to the toolbox of a policymaker, who is now not only in charge of distributing the initial permits but also allowed to shift initial private endowments. We consider the efficiency of permits market under the new expanded policy space. In particular we would like to know how the added authority to the policymaker would change (if at all) proposition 3 of section 5.5.

Mathematically, a social planner is to choose an abatement target $\tilde{a}$, its initial distribution $\{\tilde{a}_i\}$, and a reallocation of initial private endowments $\{\tilde{Y}_i\}$ such that

$$\sum_i \tilde{a}_i = \tilde{a} \quad \text{and} \quad \sum_i \tilde{Y}_i = \bar{Y}$$

and each country maximizes its own utility

$$\max u_i(c_i, a) \quad \text{subject to} \quad a_i = \Phi_i(\tilde{Y}_i - y_i), c_i - y_i = (a_i - \tilde{a}_i)p_e, \qquad (5.16)$$

the same setting as in section 5.5. The necessary conditions for the maximization of utilities, of course, are also the same as before:

$$MC_i = p_e + MRS_i, \qquad \forall\, i.$$

The new permits market equilibrium $(\{\hat{c}_i\}_{i=1,\ldots,I}, \{\hat{y}_i\}_{i=1,\ldots,I}, \{\hat{\pi}\}_{i=1,\ldots,I}, \hat{p}_e)$ solves problem (5.16) for each country and meets the market-clearing condition $\Sigma_{i=1}^{I} c_i = \Sigma_{i=1}^{I} y_i$.

Let $(\{c_i^*\}_{i=1,\ldots,I}, \{y_i^*\}_{i=1,\ldots,I}, \{\pi_i^*\}_{i=1,\ldots,I}, p_e^*)$ denote the unique Nash equilibrium associated with the fixed initial income distribution $\{\bar{Y}\}$ and the efficient initial abatement distribution $\{\theta_i^*\}$ of a^*. This is the equilibrium we discussed in the previous two sections. The next proposition establishes the connection between this equilibrium and the equilibrium $(\{\hat{c}_i\}_{i=1,\ldots,I}, \{\hat{y}_i\}_{i=1,\ldots,I}, \{\hat{\pi}\}_{i=1,\ldots,I}, \hat{p}_e)$ under the new expanded policy space.

PROPOSITION 6 Even if an authority has control over both the initial permits distribution and the private endowment reallocation, it is still the case that a permits market equilibrium will lead to an efficient allocation if and only if $\tilde{a} = a^*$, where a^* is the efficient abatement level. Furthermore, abatement levels and the reallocation of initial private endowments must satisfy $(\hat{\theta}_i - \theta_i^*) = (\tilde{Y}_i - \bar{Y}_i)/p_e^* a^*$.

PROOF. The proof is basically the same as the one to proposition 3. The only thing that is crucial to the proof of this proposition is the fact that the Pareto-efficient conditions (5.6) and (5.7) are independent of the initial private-goods distribution $\{\bar{Y}_i\}$. We provide a sketch of the proof.

Suppose that a permit market equilibrium $(\{\hat{c}_i\}_{i=1,\ldots,I}, \{\hat{y}_i\}_{i=1,\ldots,I}, \{\hat{\pi}\}_{i=1,\ldots,I}, \hat{p}_e)$ exists. If this equilibrium allocation is Pareto efficient, we must have $MC_i = MC_j$, which is true only if $MRS_i = MRS_j$ because $MC_i = p_e + MRS_i$.

Now let us go back to the Pareto-efficient frontier. By lemma 4 we know that conditions $MC_i = MC_j$ and $MRS_i = MRS_j$ would be met simultaneously by only one PE allocation $(\{c_i^*\}_{i=1,\ldots,I}, \{y_i^*\}_{i=1,\ldots,I}, \{\pi_i^*\}_{i=1,\ldots,I}, a^*)$, which means that any equilibrium that leads to an efficient allocation must have the initial abatement assignments $\{\tilde{a}_i\}$ sum up to a^*.

We are left to show that the equilibrium $(\{\hat{c}_i\}_{i=1,\ldots,I}, \{\hat{y}_i\}_{i=1,\ldots,I}, \{\hat{\pi}_i\}_{i=1,\ldots,I}, \hat{p}_e)$ does exist. Let $\tilde{a} = a^*$, $\hat{\pi}_i = \pi_i^*$, and $\hat{c}_i = c_i^*$. Clearly, $\{\hat{y}_i\}$ and $\hat{p}$ are uniquely determined by

$$\hat{a}_i = a_i^* = \Phi_i(\tilde{Y}_i - \hat{y}_i) = \Phi_i(\bar{Y}_i - y_i^*)$$

and

$$\hat{p}_e = MC_i(\hat{a}_i) - MRS_i(\hat{a}, \hat{c}_i) = MC_i(a_i^*) - MRS_i(a^*, c_i^*) = p_e^*.$$

Finally, the initial assignment of abatement levels is also unique because

$$\begin{aligned}\hat{\theta}_i &= \hat{\pi}_i - \frac{(\hat{c}_i - \hat{y}_i)}{\hat{p}_e\,\hat{a}} \\ &= \left[\theta_i^* + \frac{(c_i^* - y_i^*)}{p_e^*\,a^*}\right] - \frac{c_i^* - (\tilde{Y}_i - \bar{Y}_i + y_i^*)}{p_e^*\,a^*} = \theta_i^* + \frac{\tilde{Y}_i - \bar{Y}_i}{p_e^*\,a^*}.\end{aligned}$$

Rewrite the last equality to get $(\hat{\theta}_i - \theta_i^*) = (\tilde{Y}_i - \bar{Y}_i)/p_e^*\,a^*$. The proof is therefore complete. ■

There has been a suggestion that careful distribution of initial permits plus side payments might lead us to an abatement target preferred by a policymaking body. Proposition 6 should convince us that is not possible. The allocation of initial abatement and initial income transfers are two instruments that cannot be separated if a Pareto-efficient allocation of resource is to be achieved. The proposition shows that under the new expanded policy space it is still the case that efficiency would prevent any involvement of a policymaker in either choosing an abatement target or redistributing the initial wealth.

5.7 Lindahl-Pricing Equilibrium

A Lindahl equilibrium is a resource allocation and a set of individual prices $(\{c_i^*\}, \{y_i^*\}, \{\pi_i^*\}, \{p_i^*\})$ that maximizes

$$u_i(c_i, a) \quad \text{subject to} \quad a_i = \Phi_i(\bar{Y}_i - y_i),\ c_i - y_i = (a_i - \bar{a}_i)p_i \qquad (5.17)$$

for each country.

PROPOSITION 7 For any Pareto-efficient abatement allocation $(\{c_i^*\}, \{y_i^*\}, \{\pi_i^*\}, a^*)$, there is a unique allocation of initial abatement levels, such that with these as initial endowments the Lindahl-pricing equilibrium is Pareto efficient.

PROOF. The first-order conditions to problem (5.17) are

$$MC_i(a_i) = p_i + MRS_i(a, c_i). \qquad (5.18)$$

The argument of the proof is basically the following. Let $(\{c_i^*\}, \{y_i^*\}, \{\pi_i^*\}, \{p_i^*\})$ be an arbitrary Pareto-efficient allocation. At this allocation marginal cost, $MC_i(a_i^*)$, and marginal rate of substitution, $MRS_i(a^*, c_i^*)$, for each country are known. Then equation (5.18) uniquely defines country-specific price p_i^*. Finally, the initial abatement assignments $\bar{a}_i$ are determined uniquely by the budget equation, given a_i^*, y_i^*, c_i^*, and p_i^*. The allocation and prices thus constructed, $(\{c_i^*\}, \{y_i^*\}, \{\pi_i^*\}, \{p_i^*\})$, consist of a Lindahl equilibrium.

■

Notice that this proposition says that there is a unique relationship between a Pareto-efficient allocation and the initial abatement assignments that results in that Pareto-efficient allocation as a Lindahl equilibrium. Because in general there is an infinite number of Pareto-efficient allocations $(\{c_i^*\}, \{y_i^*\}, \{\pi_i^*\}, a^*)$ pointing to the same efficient abatement level a^*, there are as many ways of assigning initial abatement levels (with the sum a^*) that are compatible with Pareto efficiency using Lindahl markets with personalized prices.

5.8 An Extension: *M* Private Goods

The extension from one private good to M private goods is not a simple matter. Lemma 2, which was used in the proofs of almost all previous results, is not valid anymore. Remember in the case of one private good that production efficiency conditions alone determine the unique relationship between a Pareto-efficient total abatement level $\overline{a}$ and countries' production levels y_i; this is so because we have $(I - 1)$ independent equations $MC_i[a_i(y_i)] = MC_j[a_j(y_j)]$, $\forall\, i \neq j$, and $(I - 1)$ unknowns[11] $(y^1, y^2, ..., y_{I-1})$. With M private goods this is not true anymore: Production efficiency conditions $MC_{i,l}[a_i(\{y_{i,l}\})] = MC_{j,l}[a_j(\{y_{j,l}\})]$, $\forall\, i \neq j$, $\forall\, l$, offer $(I - 1)M$ independent equations, but we have $(IM - 1)$ unknowns; therefore, for $I, M \geq 2$, $y_{i,l}$ could not be determined in the same way as in the case of one private good.

Also the addition of more private goods prevents us from giving a simple proof of the existence of competitive equilibrium. These difficulties force us to take a different approach. In a reversal of the previous approach, we study the Nash equilibrium first and show that under certain regularity conditions there exists a finite number of initial abatement levels, for each of which there exists a finite number of ways of distributing the permits, such that the Nash equilibria under those initial arrangements lead to Pareto-efficient outcomes. We next show that the regularity condition for the existence of a Nash equilibrium is also sufficient for the existence of competitive equilibria. Knowing the existence of an equilibrium, propositions 1 and 2 are then revised for M private goods.

For easy reference we produce the M-private-goods version of Pareto-efficiency conditions. Basically, we have to solve problem (5.4) again. The two constraints to the optimization problem remain the same, except that this time we have to replace the scalars c_i and y_i by vectors $c_i = (c_{i,1}, c_{i,2}, ..., c_{i,M})$ and $y_i = (y_{i,1}, y_{i,2}, ..., y_{i,M})$.

Without difficulty, we arrive at the following necessary conditions for Pareto efficiency:

$$MC_{i,l}(a_i) = MC_{j,l}(a_j), \qquad \forall\, i \neq j \quad \text{and} \quad \forall\, l, \tag{5.19}$$

$$\frac{MRS_{i,l}(c_i,\ a)}{MRS_{i,k}(c_i,\ a)} = \frac{MRS_{j,l}(c_j,\ a)}{MRS_{j,k}(c_j,\ a)} \quad \text{for } \forall\, i \neq j,\ \forall\, l \neq k, \tag{5.20}$$

$$\sum_{i=1}^{I} MRS_{i,l}(a,\ c_i) = MC_{j,l}(a_j) \quad \text{for any } j \text{ and } \forall\, l. \tag{5.21}$$

These conditions are essentially the same as conditions for the case of one private good other than the addition of the new condition (5.20), which equalizes marginal rates of substitution between the M private goods across countries.

[11] If $\overline{a}$ and $a_1, a_2, ..., a_{I-1}$ are known, then a_I^* is automatically determined.

Nash Equilibrium

With proper modification and derivations, the M-private-good version of the first-order conditions to problem (5.9) is found to be:

$$MC_{i,l}(a_i) = \frac{p_e}{p_l} + MRS_{i,l}(c_i, \bar{a}), \qquad \forall\, i \neq j \quad \text{and} \quad \forall\, l, \tag{5.22}$$

$$\frac{MRS_{i,l}(c_i, \bar{a})}{MRS_{i,k}(c_i, \bar{a})} = \frac{p_e/p_l}{p_e/p_k} \qquad \forall\, i, \forall\, k \neq l. \tag{5.23}$$

Again, a new condition (5.23) is added, stating that the marginal rates of substitution between private consumption goods must equal their market price ratio. The M-private-good version of budget constraint (5.8) now becomes

$$\sum_{l=1}^{M} (c_{i,l} - y_{i,l})p_l = (a_i - \bar{a}_i)p_e, \qquad \forall\, i. \tag{5.8$'$}$$

Rewrite market clearance condition (5.9):

$$\sum_{i=1}^{I} c_{i,l} = \sum_{i=1}^{I} y_{i,l}, \qquad \forall\, l. \tag{5.9$'$}$$

Denote by p the relative price vector $(p_1/p_e, p_2/p_2, ..., p_l/p_e)$. We have the following lemma.

LEMMA 5 Assume that a Nash equilibrium (c_i^*, y_i^*, p^*) exists. The following two conditions are necessary and sufficient for the equilibrium to achieve Pareto efficiency:

(i) $\sum_{i=1}^{I} MRS_{i,k}(c_i^*, \bar{a}) = MC_{j,k}(a_i^*)$ for some j and k,

(ii) $MRS_{i,l}(c_i^*, \bar{a}) = MRS_{j,l}(c_j^*, \bar{a})$ for some l and $\forall\, i \neq j$.

PROOF. *Sufficiency.* By (5.23), clearly Pareto-efficiency condition (5.20) is always satisfied at the equilibrium. Next we show that (5.22), (5.23), and condition (ii) of lemma 5 imply the Pareto-efficiency condition (5.19). From (5.22) and (5.23) we have

$$\begin{aligned}\frac{MC_{i,l}(a_i)}{MC_{i,k}(a_i)} = \frac{p_e/p_l}{p_e/p_k} = \frac{MRS_{i,l}(\bar{a}, c_i^*)}{MRS_{i,k}(\bar{a}, c_i^*)} \\ = \frac{MC_{j,l}(a_i)}{MC_{j,k}(a_i)}, \qquad \forall\, i \neq j \quad \text{and} \quad \forall \text{l}.\end{aligned} \tag{5.24}$$

Therefore, if $MC_{i,l}(a_i^*) = MC_{j,l}(a_j^*)$ holds for $\forall\ i \neq j$ and for some l, then it holds for all $l = 1, ..., M$. However, this is exactly what we can get from (5.22) and condition (ii) of lemma 5:

$$\frac{MC_{i,l}(a_i)}{MC_{j,l}(a_i)} = \frac{MRS_{i,l}(\overline{a},\ c_i^*) + \dfrac{p_e}{p_l}}{MRS_{j,l}(\overline{a},\ c_j^*) + \dfrac{p_e}{p_l}} \quad \text{for some } l \quad \text{and} \quad \forall\ i \neq j.$$

Finally, we need to show that (5.21) is also satisfied. This is trivial.

Necessity

We show that if any part of conditions (i) or (ii) does not hold, then the equilibrium cannot achieve Pareto efficiency. It is obvious that if (i) does not hold, then Pareto-efficiency condition (5.21) will be violated. Now consider condition (ii). Suppose that the Nash equilibrium does achieve Pareto efficiency even though $MRS_{i,l}(c^*, \overline{a}) \neq MRS_{j,l}(c_{j,l}^*, \overline{a})$ for some l and some $\forall\ i \neq j$. This immediately leads to contradiction. By (5.22), if $MRS_{i,l}(c^*, \overline{a}) \neq MRS_{j,l}(c_{j,l}^*, \overline{a})$ for some l and some $\forall\ i \neq j$, then $MC_{i,l}(c^*, \overline{a}) \neq MC_{j,l}(c_{j,l}^*, \overline{a})$ for some l and some $\forall\ i \neq j$ contradicting (5.19). ■

We next show that there exist abatement levels such that with proper distribution of those levels as initial abatement endowments Nash equilibria achieve Pareto efficiency. Consider an initial abatement target $\overline{a}$ and an allocation rule $\{\theta_i\}$. According to lemma 5, a Nash equilibrium (c_i^*, y_i^*, p^*), if it exists, achieves Pareto efficiency if the initial $\overline{a}$ and $\{\theta_i\}_{i=1,\dots,I}$ are properly chosen such that conditions (5.22) to (5.24) and conditions (i) and (ii) of lemma 5 are simultaneously met. These conditions consist of $2(I \times M) + I + M$ independent equations. Note that we have the same number of unknowns, $\{c_{i,l}\}$, $\{y_{i,l}\}$, $\{\overline{a}_i\}$, and $\{p_e/p_l\}$. Denote the $2(I \times M) + I + M$ unknowns by vector x and denote the previous mapping from $\Re^{2(I\times M)+I+M} \to \Re^{2(I\times M)+I+M}$ by function Ψ. The following regularity condition on Ψ is assumed following a similar assumption in CHS.

Regularity condition. The matrix of first partial derivatives of the function Ψ has full rank.

We now provide the M-private-good version of proposition 3.

PROPOSITION 3′ Assume the *regularity condition.* At no more than a finite number of total abatement levels, and with at most a finite number of ways of distributing them among the countries as their initial endowments, could Nash equilibria lead to Pareto-efficient allocations.

PROOF. The proof part is actually easy. We need only show that the number of equilibrium points to the equation system $\Psi(x) = 0$ is finite. Because the mapping $\Psi : \Re^{2(I\times M)+I+M} \to \Re^{2(I\times M)+I+M}$ is defined on a compact set in $\Re^{2(I\times M)+I+M}$ and by regularity condition the matrix of first partial derivatives of the function Ψ has full rank, equation system $\Psi(x) = 0$ has at most a finite number of solutions x. This proves proposition 3′. ■

Competitive equilibrium

The extension from one private good to M private goods for the case of Nash equilibrium is the difficult part of this section. The rest becomes easy. Again reproduce the two first-order conditions for competitive equilibrium in the permit market:

$$MC_{i,l}(a_i) = \frac{p_e}{p_l} \qquad \forall\, i,\ \forall\, l, \tag{5.25}$$

$$\frac{MRS_{i,l}}{MRS_{i,k}} = \frac{p_e/p_l}{p_e/p_k} \qquad \forall\, i,\ \forall\, k \neq l. \tag{5.26}$$

The following lemma should be compared with lemma 3.

LEMMA 3′ Assume the regularity condition. Further assume that $\overline{a}$ and $\{\theta_i\}_{i=1,\ldots,I}$ are given. There exists at most a finite number of competitive equilibria $[c_i^*(\{\theta_i\}, \overline{a}], y_i^*[\{\theta_i\}, \overline{a}], p^*[\{\theta_i\}, \overline{a})]$.

PROOF. The logic of this proof is the same as the one to proposition 3′. Here we have unknowns $c_i^*(\{\theta_i\}, \overline{a})$, $y_i^*(\{\theta_i\}, \overline{a})$, $p^*(\{\theta_i\}, \overline{a})$, a total of $[2(I \times M) + M]$. How many equations do we have? The same number: condition (5.25) consists of $I \times M$ equations, (5.26) consists of $I \times (M - 1)$, and (5.24) and (5.25) provide additional $I + M$ equations. Denote the $[2(I \times M) + M]$ unknowns by z and the mapping from $\Re^{2(I\times M)+M}$ to $\Re^{2(I\times M)+M}$ by $\Gamma(z)$. Note that the matrix of first partial derivatives of the function Γ is a submatrix of Ψ. Therefore, the regularity condition on Ψ

implies that the matrix of first partial derivatives of the function $\Gamma(z)$ also has full rank. This ends the proof. ■

Next we show that proposition 1, with small modifications, holds for M private goods. Once again let us inspect the two sets of necessary conditions. We see that the permits market equilibrium conditions (5.25) and (5.26) imply Pareto-efficiency conditions (5.19) and (5.20). However, as before, the Pareto-efficiency condition (5.21) generally will not be automatically satisfied by a competitive equilibrium allocation.

PROPOSITION 1′ Assume the *regularity condition,* and further assume that $\bar{a}$ is given. A distribution of initial abatement $\{\theta_i\}_{i=1,\ldots,I}$ associated with $\bar{a}$ leads to a Pareto-efficient allocation of resources at an equilibrium if and only if that equilibrium allocation $[c_i^*(\{\theta_i\}, \bar{a}), y_i^*(\{\theta_i\}, \bar{a})]$ satisfies condition (i) of lemma 5, or

$$\sum_{i=1}^{I} MRS_{i,k}(c_{i,k}^*, \bar{a}) = MC_{j,k}(a_i^*) \quad \text{for some } j \text{ and some } k. \tag{5.27}$$

PROOF. An equilibrium must satisfy (5.25) and (5.26). Basically, we need to show that if an equilibrium further satisfies (5.27), then the Pareto-efficiency condition (5.21) is also satisfied. From equations (5.25) and (5.26) we have

$$\frac{MRS_{i,k'}}{MRS_{i,k}} = \frac{p_k}{p_{k'}} = \frac{MC_{i,k'}}{MC_{i,k}} \quad \text{for all } \mathrm{k}' \neq k \text{ and all } i,$$

from which we easily have

$$\sum_{i=1}^{I} MRS_{i,k'} = \sum_{i=1}^{I}\left(MRS_{i,k}\frac{p_k}{p_{k'}}\right) = \frac{MC_{j,k'}}{MC_{j,k}}\sum_{i=1}^{I} MRS_{i,k} \quad \text{for any } j \text{ and } k' \neq k,$$

or

$$\frac{\sum_{i=1}^{I} MRS_{i,k}}{MC_{j,k}} = \frac{\sum_{i=1}^{I} MRS_{i,k'}}{MC_{j,k'}} \quad \text{for any } j \text{ and any } k' \neq k.$$

Clearly, if $\Sigma_{i=1}^{I} MRS_{i,k} = MC_{j,k}$, then for any $k' \neq k$ we also have $\Sigma_{i=1}^{I} MRS_{i,k'} = MC_{j,k'}$. This completes the proof. ■

There is one crucial point to be made concerning the issue of multiple equilibria. As lemma 3′ states, each initial $\overline{a}$ and its distribution $\{\theta_i\}$ is associated with a finite number of equilibria. We do not know in advance which equilibrium will be realized. It is of course sufficient if all the equilibrium allocations associated with the pair $(\overline{a}, \{\theta_i\})$ satisfy condition (5.27). In that case the total number of constraints on the distributions $\{\theta_i\}$ would be the number of equilibria plus one. The additional constraint, of course, comes from $\Sigma_i \theta_i = 1$. Because we do not know how many equilibria are associated with each $(\overline{a}, \{\theta_i\})$, we are unable to make a definite statement about what $\{\theta_i\}$ would lead to a Pareto-efficient allocation of resources. For the special case of unique equilibrium, the number of constraints on the initial distribution of $\{\theta_i\}$ would be two: one from (5.27) and the other $\Sigma_i \theta_i = 1$.

We have therefore confirmed that proposition 1 indeed holds whether we have one or *M* private goods.

5.9 Concluding Remarks

A global emission permits market has been favored as one of the mechanisms that can effectively control greenhouse gas emissions. A practical implementation issue for such a system to "succeed" is how the emission permits will be allocated between participating countries. Equity is a central issue here. In addition, economists generally would not consider a system successful unless it is efficient. Efficiency and equity are therefore at the center of the debate as to whether a global permit market is preferable to other mechanisms. It has been shown in CHS that the two are inseparable. Here we further refine the CHS findings. We provide necessary and sufficient conditions on the distribution of initial permits such that under those conditions the resulting market equilibrium attains a Pareto-efficient outcome. Furthermore, we extend the CHS model to allow for strategic behavior. We show under this new behavioral assumption on the part of emission abatement participants that the choices of a policymaking body are very limited: At no more than a finite number of total abatement levels and for each abatement level with at most a finite number of ways of distributing the permits among the countries as their initial endowments could the resulting Nash equilibrium lead to Pareto-efficient allocations.

Our equilibrium analysis of countries' initial abatement assignments and permits trading positions provide some topics for further debate or justification. For example, why should it be that, between two countries with the same initial endowments in private goods and the same abatement technology, the country that is more environment conscious is assigned a lower abatement level? Is our efficiency justification sufficient to justify for it? To answer these questions, further research is required.

The model presented here is a static one. We hope to extend it to a dynamic framework in the future. Some preliminary work with two time periods has already be done by Eyckmans, Proost, and Schokkaert [11].

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