Chapter 3

Equity and Efficiency in Environmental Markets: Global Trade in Carbon Dioxide Emissions

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3.1 Equity, Efficiency, and Carbon Dioxide Abatement

This chapter addresses a topical issue: the creation of a global market for carbon dioxide (CO_2) emission permits.[1] The recent adoption in the Kyoto Protocol of an ambitious target for global CO_2 emission has focused attention on policy instruments for achieving this goal. In addition, increasing awareness of the economic burden of environmental protection has produced an interest in market-based policy instruments that can minimize detailed government intervention. As a result markets for emission rights are today the approach of choice of the U.S. administration.[2]

This chapter is based on Chichilnisky, G., Heal, G., and Starrett, D. "International Markets with Emissions Rights of Greenhouse Gases: Equity and Efficiency," Center for Economic Policy Research Publication No. 81, Stanford University, Fall 1993.

[1] The atmospheric concentration of CO_2 has become a matter of international concern. It is generally recognized that it has the capacity to change the global climate in ways that are potentially harmful and irreversible. For a review, see Chichilnisky and Heal [3] and Chichilnisky et al. [8]. Consequently, countries at the 1992 Earth Summit in Rio de Janeiro agreed to cut back CO_2 emissions to their 1990 levels by the end of the twentieth century. This policy could easily cost several percent of GNP (see Weyant [27]). In conformity with the conclusions of the Earth Summit, the U.S. administration has recently made a tentative move in the direction of capping CO_2 emissions in industrial countries.

[2] According to a statement by Tim Wirth, U.S. assistant secretary of state for global affairs, at the 1996 Berlin Conference of the Parties of the Framework Convention on Climate Change.

We show that a market for emission permits has an important characteristic not previously noted, a characteristic that has significant economic and political implications. When the level of emissions affects utilities, there is an unexpected link between equity and efficiency: The initial distribution of property rights or emission permits determines whether a competitive global CO_2 permit market will operate efficiently.[3] Prior to now it has been generally assumed that the manner in which emission permits are initially distributed will not affect the efficiency of the market.[4] We show here that of all the many possible ways of distributing a given total of emission rights, very few are compatible with efficient markets. In this case equity and efficiency are not orthogonal, as in the first and second theorems of welfare economics for standard competitive markets. How does this happen?

The key to this result is the fact that the atmospheric concentration of CO_2 is a privately produced public good, privately produced but affecting the utility levels of all people. The reason is that CO_2 mixes thoroughly in the atmosphere, leading to a uniform concentration over the globe. Therefore, we have a global public good. People or regions cannot choose their concentration levels independently. However, the concentration is determined by every individual who runs a car or a heating furnace and by every firm operating transportation or burning fuel in any other way.[5] Therefore, we have a privately produced public good. The fact that CO_2 concentration is a privately produced public good affecting the welfare levels of individuals leads to the equity-efficiency interaction. As noted, everyone has de facto to consume the same CO_2 concentration. For efficiency this common level must be what they demand, given prices and their incomes. In summary, for agents to demand freely the same amounts of CO_2 at an equilibrium requires a particular choice of the distribution of income.

Similar points were made in Chichilnisky [2] and Chichilnisky and Heal [4],[6] where this simple observation was shown to have other far-reaching consequences. In particular these papers establish that the equalization of marginal abatement costs across countries is neither sufficient nor necessary for Pareto

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[3] The term *efficiently* here is used in the standard economic sense of "so as to attain Pareto efficiency."

[4] It will of course affect the distribution of income resulting from the operation of the market. This is the original Coase [9] position: that whatever the initial distribution of permits, trading rights can bring about a Pareto-efficient allocation of resources. In fact a stronger claim is sometimes made: that the equilibrium allocation of resources is not affected by the initial distribution of permits. Clearly, the conditions for this stronger claim to be true are very restrictive indeed—a total absence of income effects; see Milgrom and Roberts [22], chapter 2.

[5] Carbon dioxide, a public bad, is a by-product of the consumption and production of private goods.

[6] There is also an early discussion of closely related issues in Laffont [19] and Eyckmans et al. [13].

efficiency: Pareto-efficient allocations may have different marginal costs. Here we show that this line of argument, when developed further, implies that efficiency and distribution cannot be separated in environmental markets. Efficiency requires an appropriate distribution of property rights. The fact that many distributions of property rights lead to inefficient outcomes allows us to construct an example of a two-region world in which a transfer of property rights from the North to the South, accompanied by a decrease in the total of emission permits, leaves both regions better off.

Finally, we investigate the extent to which an equilibrium concept related to that of Lindahl is the appropriate concept in permit markets. There is a simple reason that this might be so: A Lindahl equilibrium is the only market equilibrium known to lead to Pareto efficiency with public goods.[7] As a permit market is a market that determines the production of public goods, we might therefore expect that efficiency would require the key feature of a Lindahl equilibrium, namely, a multiplicity of prices, in fact one price per pair of traders. In a Lindahl equilibrium each producer of a public good is paid for her production by each consumer, and the per unit payment typically varies from consumer to consumer. Therefore, relative to the framework of a Lindahl equilibrium, a permit market as formalized here is an "incomplete market" because everyone pays the same price for the permits. This can be interpreted as assuming that the "individualized" markets between buyers and sellers are missing. Our main result shows that, in a certain sense, it is possible to compensate for the absence of individualized markets by reallocating property rights in tradable permits.[8]

3.2 Efficiency and International Emissions

Following the model set out in Chichilnisky [2] and developed further in Chichilnisky and Heal [4], we consider a world economy with I regions, $I \geq 2$, indexed by $i = 1, ..., I$. Each region has a utility function u_i, which depends on its consumption of a vector of private goods $c_i = (c_{i,1}, c_{i,2}, ..., c_{i,M})$, where M is the number of private goods (indexed by m), and also on the quality of the world's atmosphere, a, which is a public good.[9] The quality of the atmosphere a can be thought of as a measure of abatement. It could be measured by, for example, the reciprocal or the negative of the concentration of CO_2: The more abatement there is, the lower is this concentration. The concentration of CO_2

[7] See Foley [14].

[8] The dimensionality of the space of permit allocations equals that of the space of Lindahl prices needed to complete the market, so that the two approaches are mathematically equivalent.

[9] Formally, $u_i(c_i, a)$ measures welfare, where $u_i: \Re^{M+1} \to \Re$ is a continuous, strictly concave and increasing function. It is assumed to be twice continuously differentiable.

is "produced" by emissions of carbon, which are positively associated with the levels of production of private goods. Let y_i be a vector $(y_{i,m})$ in R^M giving the production levels of the M private goods in country i. Then

$$a = \sum_{i=1}^{I} a_i,\ a_i = \Phi_i(y_i) \quad \text{for each country } i = 1, ..., I, \quad \text{and} \quad \frac{\partial \Phi_i}{\partial y_{i,m}} < 0 \qquad \forall i \tag{3.1}$$

The production functions or abatement functions Φ_i are continuously differentiable and strictly concave and show the trade-off between the level of abatement or quality of the atmosphere and the output of consumption.[10] An allocation of consumption and abatement across all countries is a vector

$$(c_1, a_1, ..., c_I, a_I) \in \Re^{(M+1)I},$$

as for each of the I regions there are M private goods and one level of abatement. An allocation is feasible if it satisfies constraint (3.1), and the condition that the total consumption of each private good worldwide be equal to the total production, that is,

$$\sum_{i=1,...,I} c_i = \sum_{i=1,...,I} y_i \tag{3.2}$$

Constraint (3.2) allows private goods to be transferred freely between regions; that is, it allows unrestricted lump-sum international redistributions. This is a rather strong assumption that gives a full first-best solution. It is not of course equivalent to modeling free trade in international markets because the latter requires that each region trade within its budget: each region must satisfy a balance of payments condition.[11]

3.2.1 Characterization of Pareto Efficiency — In this section we provide a characterization of Pareto-efficient allocations. This section does not address

[10] We can suppose that the functions Φ_i embody information about countries' initial endowments of goods. By assuming strict concavity, we are bypassing the possible nonconvexities associated with externalities (Starrett [24]).

[11] See Chichilnisky and Heal [5]. International trade between regions would require that

$$\forall i, \qquad (c_i - y_i)p = 0, \tag{3.3}$$

where $p \in \Re^m$ is a world price vector. This condition requires the value of the difference between consumption and production to be zero at world prices, which implies that for each region the value of goods that are imported and for which consumption exceeds production equals the value of goods that are exported and for which production therefore exceeds consumption.

any institutional framework, as it does not presume any structure, such as emission markets or emission taxes. It describes the conditions that any resource allocation must satisfy if it is efficient, whatever the institutional structure through which it is implemented.

Lump-Sum Transfers

An allocation is called *feasible with lump sum transfers* if it satisfies constraints (3.1) and (3.2). Such an allocation $(c_1^*, a_1^*, ..., c_I^*, a_I^*) \in R^{(M+1)I}$ is *Pareto efficient* if there is no other feasible allocation at which every region's utility is at least as high, and one's utility is strictly higher.[12] It is immediate therefore that a Pareto-efficient allocation solves the following problem:

$$\max\ u_i(c_i, a) \quad \text{subject to} \quad u_k(c_k, a) = N_k\ \forall k \neq i,\ k = 1, ..., I,$$
$$\textstyle\sum_{i=1}^{I} y_{i,m} = \sum_{i=1}^{I} c_{i,m}\ \forall m,$$
$$a_i = \Phi_i(y_i), \quad \text{and} \quad \textstyle\sum_i a_i = a. \tag{3.4}$$

Here N_k is a utility level specified for region k.[13]

To solve problem (3.4) we can write out the corresponding Lagrangian

$$L = u_i\left(c_i, \sum_{i=1}^{I} \Phi_i(y_i)\right) + \sum_{k=1,...,I,k\neq i} \lambda_k\left(u_k\left(c_k, \sum_{i=1}^{I} \Phi_i(y_i)\right) - N_k\right)$$
$$+ \sum_{m=1}^{M} \theta_m\left(\sum_{i=1}^{I} y_{i,m} - \sum_i c_{i,m}\right),$$

where a has been replaced by $\Sigma_i\, \Phi_i(y_i)$ in view of (3.1). Differentiating L with respect to the components of c_i and y_i and equating to zero gives the first-order conditions for efficiency (3.5) and (3.6):

$$\overbrace{\frac{\partial u_i}{\partial c_{i,m}} = \lambda_k \frac{\partial u_k}{\partial c_{k,m}} \qquad \forall m = 1, ..., M \quad \text{and} \quad \forall k \neq i,}^{\text{equal marginal valuations of consumption}} \tag{3.5}$$

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[12] A Pareto-efficient allocation can be characterized as a solution to the problem of maximizing the utility of a designated region, subject to the others all reaching prescribed utility levels. The solutions of this problem (as the prescribed utility levels vary over all feasible values) describe the utility possibility frontier.

[13] Observe that the second line of this problem allows unrestricted international lump-sum redistribution. Worldwide consumption has to equal worldwide production, with no region-by-region balanced budgets required.

where i is the designated region whose utility is being maximized, λ_k is a Lagrange multiplier associated with the constraint that region k should reach a specified welfare level, and

$$\overbrace{\frac{\partial \Phi_i}{\partial y_{i,m}} = \frac{-\frac{\partial u_i}{\partial c_{i,m}}}{\Sigma_k \lambda_k \frac{\partial u_k}{\partial a}} \; \forall m, \quad \text{and for } k \neq i, \quad \frac{\partial \Phi_k}{\partial y_{k,m}} = \frac{-\lambda_k \frac{\partial u_k}{\partial c_{k,m}}}{\Sigma_k \lambda_k \frac{\partial u_k}{\partial a}} \; \forall m.}^{\text{Lindahl-Bowen-Samuelson condition}} \tag{3.6}$$

Each of these systems of equations has a simple interpretation. The first system, (3.5), requires that for any good m the marginal social value of consumption be the same for all regions i. We refer here to the "marginal social value of consumption by region i" because the marginal utilities of consumption are weighted by the terms λ_k, which represent the shadow price or social value of utility in region k. The second set of equations, (3.6), is a slight modification of the conventional Lindahl-Bowen condition, popularized by Samuelson. It requires that the marginal rate of transformation between the public good and a private good be equal to the sum of the marginal rates of substitution. (See also chapter 13 for a detailed analysis of efficiency conditions.)

Without Lump-Sum Transfers

If we restrict international lump-sum redistributions, the corresponding characterization of (constrained) Pareto efficiency is different. For example, if we model an autarchic world where in each region consumption is required to equal production, the second line of the problem (3.4) is dropped and the vector y_i in the third line replaced by c_i. In this case the necessary conditions for Pareto efficiency are just (3.6). Condition (3.5) is no longer required.

Should Marginal Costs Be Equal?

Note that the marginal cost of abatement in region i in terms of good m is just the reciprocal of the marginal productivity with respect to m of the function Φ_i:

$$MC_{i,m}(a_i) = -\frac{1}{\frac{\partial \Phi_i}{\partial y_{i,m}}}. \tag{3.7}$$

PROPOSITION 1[14] At a Pareto-efficient allocation $(c_1^*, a_1^*, ..., c_I^*, a_I^*)$, in each country the marginal cost of abatement $MC_i(a_i^*)$ in terms of private good m is inversely proportional to the marginal valuation of the private good m, $\lambda_i \partial u_i / \partial c_{i,m}$. In particular, at a Pareto efficient allocation, the marginal costs will be equal across countries if and only if the marginal valuations of the private goods are equal; that is, for each good m, $\lambda_i \partial u_i / \partial c_{i,m}$ is independent of i.

It follows that with lump-sum transfers, as represented by constraint (3.2), marginal costs will always be equalized, as private goods can always be shifted between countries by lump-sum redistributions to equate their marginal valuations. However, if each country is required to consume what it produces or is required to trade internationally subject to a standard balance of trade constraint, this is not true.[15] Therefore, in general equalization of marginal costs across countries is not necessary for efficiency.

3.3 International Emission Markets

In section 3.2 we characterized in equations (3.5) and (3.6) allocations that are Pareto efficient in an institution-free framework as well as those in which each region consumes what it produces.

Next we introduce an institutional framework: an international market for tradable permits. The aim is to investigate the first-best efficiency of the equilibria in this market. To model a policy-relevant situation, assume that the initial distribution of emission permits is the only variable used to address distributional issues.[16] Each region is given an initial endowment of permits to emit E_i units of CO_2, where $\Sigma_i E_i = E^*$, the desired level of total emissions. Regions trade these and behave as price takers in a market in which there is a single price p_e for a permit to emit one unit.

If the number of units of CO_2 emitted exceeds the number of permits a region has, the region must buy the difference in the permit market. Otherwise, it can sell excess permits and use the proceeds to buy private goods at prices p_l. A region therefore maximizes its utility $u_i(c_i, a)$ subject to the following budget constraint:

$$\sum_{m=1}^{M} c_{i,m} p_m = \sum_{m=1}^{M} y_{i,m} p_m - p_e(E_i + a_i). \qquad (3.8)$$

[14]Chichilnisky and Heal [4] established the following proposition in the case of one private good. The extension to the present case, which differs only in having many private goods is immediate.

[15]See Chichilnisky and Heal [5].

[16]In particular, unrestricted lump-sum redistributions of private goods are not possible.

The difference between actual emissions e_i and target emissions E_i is $e_i - E_i = e_i^N - a_i - E_i$, where e_i^N is the emission level of region i when abatement is zero.[17] The budget constraint requires that in each region the value of consumption equal the value of production plus the net revenue from the sale of permits. This can be rewritten as

$$\left(\sum_{m=1}^{M} c_{i,m} p_m - \sum_{m=1}^{M} y_{i,m} p_m\right) = -p_e(E_i + a_i). \tag{3.9}$$

The left-hand side is the difference between the value of domestic consumption and production, that is, the balance of trade. A surplus of consumption over production[18] is funded by the revenue generated by sales of permits in international markets. Conversely, a net purchase of permits in international markets has to be matched by a surplus of production over consumption and therefore a net export position.

A comparison of the balance-of-trade condition (3.9) with the actual budget constraint (3.3) suggests that controlling the initial endowments of emission rights can act as a substitute for lump-sum transfers. This point is developed later in section 3.4.

Each region seeks to maximize its utility $u_i(c_i, a)$ subject to the budget constraint (3.8) and to the production relations given in (3.1). We assume that in so doing it supposes the total level of emissions to be fixed at E^*, the desired total level. This in effect implies the existence of a credible intergovernment agency (the UNFCC, for example) that sets and implements global emission targets.[19]

3.3.1 Market Behavior — Maximizing its welfare subject to the budget constraint (3.9), each region chooses consumption levels and abatement or emission levels to satisfy the following first-order conditions:

$$\text{MRS} = \text{price ratio, or} \quad \frac{\dfrac{\partial u_i}{\partial c_{i,l}}}{\dfrac{\partial u_i}{\partial c_{i,j}}} = \frac{p_l}{p_j}, \tag{3.10}$$

[17]For simplicity we have dropped the constant terms in e_i^N.

[18]That is, a position of net imports.

[19]An alternative, which we do not explore here, would be to look for a Nash equilibrium in countries' abatement levels. In this Nash case each country would observe the emissions of each other and then choose its optimal emission level on the assumption that these levels are fixed. This approach is developed in Heal and Lin [18] (chapter 5 in this volume). For a similar development, see Dasgupta and Heal [12], chapter 3.

and

$$\text{MRT} = \text{price ratio, or} \quad \frac{\partial \Phi_i}{\partial y_{i,l}} = -\frac{p_l}{p_e}. \qquad (3.11)$$

These are standard conditions for utility maximization subject to production and budget constraints. First-order condition (3.10) just requires that marginal rates of substitution between goods be equated to their price ratios, and (3.11) requires tangency between the production possibility frontier and an isoprofit hyperplane.

3.3.2 Market Solutions that Are Not Pareto Optimal — How do first-order conditions (3.10) and (3.11) characterizing a region's optimal market choice compare with conditions (3.5) and (3.6), which describe Pareto-efficient allocations? Condition (3.11) from regional utility maximization is the same as the Bowen-Lindahl-Samuelson condition (3.6) for the efficient provision of public goods, provided that

$$\frac{p_m}{p_e} = \frac{\dfrac{\partial u_i}{\partial c_{i,m}}}{\Sigma_{k=1}^{I} \lambda_k \dfrac{\partial u_k}{\partial a}} = \frac{\lambda_k \dfrac{\partial u_k}{c_{k,m}}}{\Sigma_{k=1}^{I} \lambda_k \dfrac{\partial u_k}{\partial a}} \qquad \forall k \neq i. \qquad (3.12)$$

Condition (3.12) can hold only if the marginal valuations of the *m*th private good, $\partial u_i / c_{k,m}$ and $\lambda_k (\partial u_k / \partial c_{k,m})$, are independent of *i* and *k*, that is, are the same for all regions.

Condition (3.5) is required for Pareto efficiency—equalization of the marginal valuation of consumption across countries—and automatically implies this. However, there is nothing equivalent to (3.5) in the solutions to the regions' optimization problems. The only other condition from each regions' own optimization problems is (3.10), which does not imply equality of marginal valuations across countries.

In brief, utility maximization subject to the budget constraint (3.8) does not lead to the conditions needed for Pareto efficiency, as illustrated in figures 3.1 and 3.3 below. The next section provides a simple geometric example illustrating this result. There is an additional requirement represented by (3.5). For the Bowen-Lindahl-Samuelson condition to hold, we need the marginal valuation of consumption to be the same in all regions; that is, $\partial u_i / \partial c_{i,m} = \lambda_k (\partial u_k / \partial c_{k,m})\ \forall m, \forall k \neq i$. This condition would of course be satisfied if there were policy instruments available to redistribute freely all resources without restriction across regions—if, for example, lump-sum redistributions were possible. In the absence of such instruments, what is required to ensure that (3.5) is met and efficiency attained in the permit market?

3.4 Equity and Efficiency in Permit Markets

Competitive permit markets do not generally lead to the conditions for Pareto efficiency because there is nothing that ensures that condition (3.5), $\partial u_i / \partial c_{i,m} = \lambda_k (\partial u_k / \partial c_{k,m})\ \forall m = 1, ..., M$ and $\forall k \neq i$, is satisfied. Now this is clearly a condition on the distribution of income or wealth. Look in more detail at the determinants of the terms $\partial u_i / \partial c_{i,m}$. As $u_i = u_i(c_i, E^*)$, where E^* is fixed, the derivatives of u_i with respect to consumption can depend only on consumption levels.[20] In the absence of policy instruments to achieve unrestricted redistributions across regions, the only variables then available for ensuring that marginal social valuations of consumption are equalized are the initial allocations of permits, and therefore only those initial permit allocations that ensure that (3.5) is satisfied will lead to Pareto-efficient allocations. We formalize this in the following and show that very few initial allocations satisfy this condition.

3.4.1 Why Distribution Matters — An intuitive explanation for the dependence of efficiency on distribution is as follows. Because we are trading a public good, everyone must consume the same amount at equilibrium, a physical requirement resulting from the fact that the gas CO_2 distributes uniformly across the world. Achieving more targets typically requires more instruments, and here the extra instruments are the distribution of emission permits or property rights. The efficient distributions of property rights are those at which there are market-clearing prices such that all regions demand freely the same level of the public good. If regions' preferences were similar, this would require similar income levels. A useful comparison is with a Lindahl equilibrium, the standard market equilibrium concept for public goods, in which the extra instruments are provided by region-specific prices. Recall that at a Lindahl equilibrium the prices for public goods will typically be different for different consumers, so that with Lindahl markets different regions would pay different prices for emission permits. In this case permit trading would not equalize marginal abatement costs across regions.

[20]These in turn depend, by the budget constraint (3.8), on prices p_m, production levels $y_{i,m}$, abatement levels a_i, and initial endowments of emission rights E_i. Once prices are given, production and abatement levels are fully determined by (3.11).

Another explanation for the significance of the distribution of property rights is as follows:

1. Trading emission permits naturally leads to the equalization of marginal abatement costs across countries. By obvious arguments each country equates the marginal cost of abatement to the price of an emission permit, which by assumption is the same for all countries (see equation [3.11]).
2. Equalization of marginal costs is efficient only if marginal social valuations of consumption are equalized (see proposition 1). Therefore, permit trading is efficient only if marginal social valuations of consumption are equalized. This can be achieved only by an appropriate redistribution of wealth.
3. The assignment of property rights brings about a redistribution of wealth. The efficient allocations of permits are those that equate marginal valuations of consumption.

3.4.2 An Example: One Private Good and Two Regions — Imagine two regions trading one private good and one public good (abatement). Figure 3.1 shows the abatement-production frontier and the preferences over combinations of public and private goods for each region. An emission level E^* has been chosen that we assume is a level associated with a Pareto-efficient allocation. Therefore, the question before us is, When can we attain this efficient allocation of resources by trading emission permits?

The total abatement level of the two regions must be $-E^*$, and because they are identical, each must produce a level of abatement of $-E^*/2$. Each region's production of the private good is now determined to be the level that corresponds to an abatement level of $-E^*/2$, so that the production points of the regions are now determined as in figure 3.1. As a result, the relative price of the public and private good is determined and is the slope of the frontier at this point. Each region's consumption of the public good abatement is the total amount of abatement produced, $A^* = -E^*$, and its consumption of the private good is determined by maximizing utility subject to the equation

$$c_i = y_i - p_e(E_i + a_i),$$

where c_i and y_i are region i's consumption and production of the single private good and p_e is the relative price of the emission permits. Here y_i, p_e, and a_i are fully determined from the total level of emissions E^* by the following chain. Total emissions E^* imply individual emissions $E^*/2$, which imply abatement levels, which imply production levels and the price of permits relative to the consumption good. Therefore, only E_i, the initial endowment of permits, is

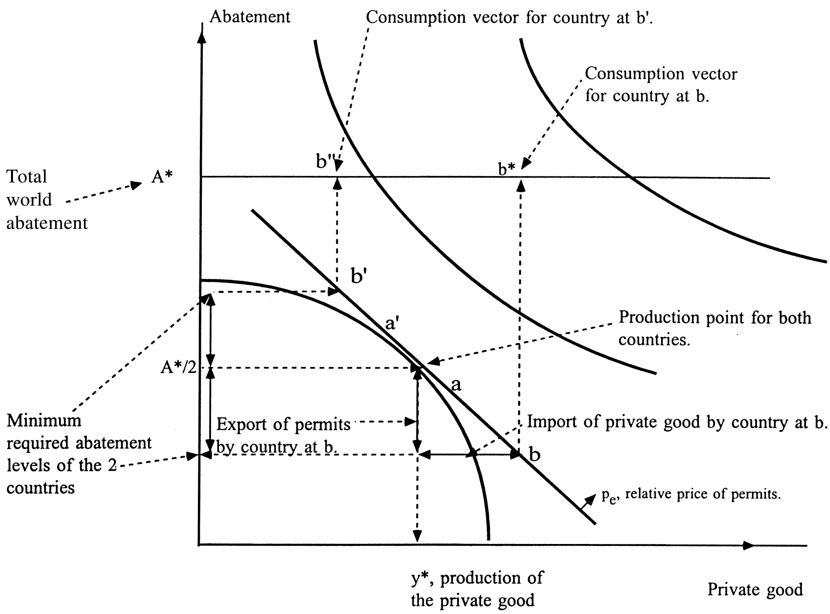

FIGURE 3.1 Only specific distributions of property rights lead to Pareto efficiency.

available to control consumption c_i. This variable must therefore be used to ensure that marginal valuations of the private good satisfy the condition (3.5) needed for Pareto efficiency. Figure 3.1 illustrates how this can be done. If both regions are given endowments of permits equal to their levels of emission, neither will trade permits, and each will consume the amount of the private good that it produces. They will consume levels of the private good given by the horizontal coordinate of the production point in figure 3.1, y^*. Their consumption of the public good abatement will be the sum of the production levels of both regions, A^*. Each region's consumption vector has a vertical coordinate equal to A^* and a horizontal coordinate equal to its consumption of the private good, namely (y^*, A^*).[21]

Consider further the case in which both regions have an initial allocation of permits equal to their production of CO_2. As they both neither import nor export the private good and so consume and produce the same amounts and also

[21] In general this is production plus imports from the sale of permits or minus exports to pay for the purchase of permits. Both are zero if countries are given endowments of permits equal to their levels of emission.

consume the same amount of the public good, their marginal valuations of the private goods must be the same.

Suppose now that condition (3.5) requires for efficiency that the marginal valuations of the private good are different, that is, that $\partial u_1/\partial c_{1,l} = \lambda_2 (\partial u_2/\partial c_{2,l}) < \partial u_2/\partial c_{2,l}$, where 1 and 2 are the two regions and l denotes the single private good. Then to satisfy (3.5) region 2's consumption of the private good has to be decreased and region 1's increased from their common production level. This can be achieved by giving region 1 an endowment of permits (b) in excess of its emissions and region 2 an endowment (b′) less than its emissions. Region 1 then increases its consumption of the private good by selling its spare permits and using the proceeds to buy the private good, whereas region 2 is forced to sell the private good to buy permits. Region 1's marginal utility of the private good will be less than region 2's, and the ratio will decrease continuously from unity as region 1's initial endowment of permits is raised above the emission level corresponding to its production of the private good (and region 2's is correspondingly reduced).

Consider the straight line p_e through the regions' production points tangent to the production frontier, as shown in figure 3.1. Each region produces a mix of abatement and private good given by the point of tangency and then trades private goods for emission permits along the line tangent to the production frontier. If it has more permits than needed (i.e., more than $E^*/2$), it will add consumption of the private good by selling permits and buying the private good along the tangency line, whose slope is the relative price of permits and the private good. As it moves along this line, its consumption of abatement remains constant.[22] However, its consumption of the private good changes. The other region will be symmetrically placed on this line relative to the production point (y^*, A^*). In this way we can reach an allocation at which all markets will clear, total emissions will be E^*, and condition (3.5) needed for efficiency will be satisfied. We can do this by picking the permit allocations and therefore consumption levels of the private good correctly. As the ratio of the regions' marginal utilities changes continuously with their initial allocations of permits, there will generally be at most a finite number of initial allocations at which the efficiency conditions hold. In this simple example, there will be just one initial distribution of permits that will lead to efficiency. This argument illustrates the following result.

PROPOSITION 2 Let E^* be the level of total emissions at a Pareto-efficient allocation of resources in the economy described in section 2 with one private

[22] It is selling surplus permits, not abatement.

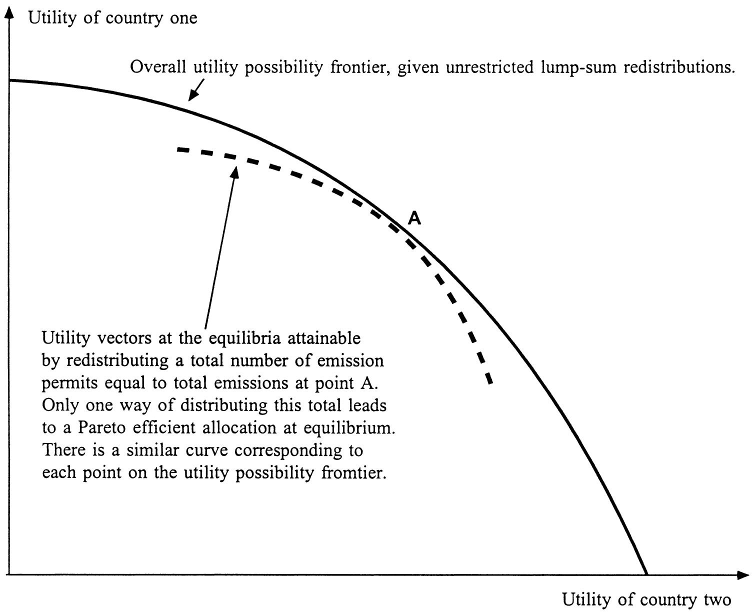

FIGURE 3.2 Redistribution of a fixed total of emission rights leads to a utility possibility curve inside the Pareto frontier.

good and two regions. Then of all possible ways of allocating the total emission E^* among the regions as initial endowments, only a subset of measure zero will lead to market equilibria that are Pareto efficient. Alternatively, almost every allocation of permits between regions will lead to inefficient outcomes.

For a proof, see the Appendix.[23]

The diagrammatic analysis illustrating proposition 2 can in fact be pushed further, as in figure 3.2. As figure 3.1 shows, each possible distribution of the total emission permits E^* between the two regions leads them to a pair of levels of consumption of the private good given by the horizontal coordinates of pairs of points, such as (a, a') or (b, b'), which are symmetrically placed on the line that is tangent to the production frontier at the production point. These pairs of points in turn give rise to consumption vectors for the public and private and

[23] The results in proposition 2 are robust. They hold not only for first-best, or Pareto, efficiency, as discussed previously, but also for efficiency subject to an arbitrary abatement constraint (see Heal [17]). In this case it is still true that only certain specific distributions of emission rights are compatible with efficiency, defined now as maximization of the sum of utilities subject to feasibility constraints and also to a politically imposed constraint on the level of emissions.

private goods, together represented by points such as b'' and b^* in figure 3.1. From figure 3.1 we can ascertain the utility levels of these points. Suppose that we plot the utility levels arising from all such possible distributions of the total E^* permits. What does this set of points look like?

We know that few points will be Pareto efficient, so that this must form a curve largely inside the utility possibility frontier, touching this frontier at a finite number of points, at most. In fact in the present two-region fully symmetric case, it is easy to see that once we have an allocation of permits that satisfies (3.5), departures from this allocation increase the difference from equality of the two sides in (3.5), so that the efficient allocation is unique. Figure 3.2 therefore illustrates the set of utility vectors associated with different allocations of the total of E^* permits and also shows the overall utility possibility frontier. Each point on the frontier corresponds to a different total emission level and therefore to a different total number of permits, and for each point on the frontier there is one way of allocating the corresponding total of permits that is efficient and gives the utility vector on the utility possibility frontier. [24]

3.4.3 Pareto-Improving Reallocations from North to South: Win-Win Solutions — A consequence of proposition 2 is that in general a competitive market in emission permits admits changes in the total and the distribution of permits that are Pareto improving, something that is of course not possible in competitive markets for private goods. Figure 3.3 illustrates such a situation.

This figure refers to two regions, called, for obvious reasons, North and South. Both are identical in production possibilities and preferences. The production frontier and two indifference curves are shown. We consider a decrease in the total number of emission permits (an increase in abatement) coupled with a transfer of permits from the North to the South and show that this can be Pareto improving for both regions simultaneously.

The initial abatement level is given by the vertical coordinate of the lower of the two solid horizontal lines and the final by that of the higher. The initial production point is therefore determined so that abatement by each region is half the initial total. Relative prices of permits and the private good are given by the slope of the production frontier at this point, and the initial permit distribution is such that the initial abatement levels of the North and South are as shown. This leads to consumption levels for the North and the South on the higher and the lower indifference curve, respectively.

Now consider a different and lower total of emission permits, one corre-

[24] Lin [20] solves analytically for the curves in figure 2 for specific utility and production functions.

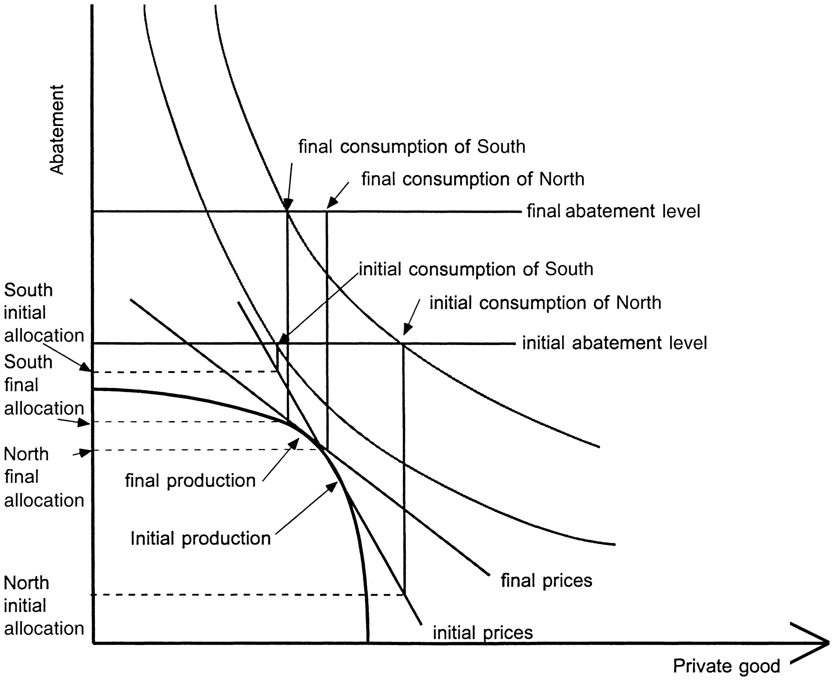

FIGURE 3.3 A redistribution of property rights from North to South can make both better off.

sponding to the higher final abatement level. Each region has to produce less of the private good and abate more, as shown by the point "final production." At the same time as the total abatement target is raised, the South's abatement target is lowered (from "South initial" to "South final"), and the North's is raised. In other words permits are transferred from North to South while the total is reduced. The new equilibrium consumption levels are as shown. Both regions are now better off, and the level of world emissions is lower.

3.4.4 The General Case — The result in proposition 2 holds for the general case, but the argument is less intuitive. Formally, we establish the following proposition:

PROPOSITION 3 Let E^* be the level of total emissions at a Pareto-efficient allocation of resources in the economy described in section 2. Assume that regions maximize utility subject to the budget constraint (3.8) given by the ability to trade emission permits. Assume furthermore that a regularity condition defined in the Appendix is satisfied. Then of all possible ways of allocating the total emission E^* among the regions as initial endowments, only a subset of measure zero will lead to market equilibria that are Pareto efficient. Alternatively, almost every allocation of permits between regions will lead to inefficient outcomes. If the inequality $(I - 1) + m \leq (I - 1) \times m$ holds, then only a finite number of ways of allocating the emission rights lead to efficiency.

The proof of this proposition is given in the Appendix. Strict concavity and the regularity assumption are needed for this result. Otherwise, one can construct counterexamples. For example, with quasi-linear preferences of the form $u_i(a) + \alpha_i c_i, \alpha_i > 0$, there might be infinitely many allocations of permits that will lead to efficient outcomes.

Although the dependence of efficiency on distribution runs quite counter to the thrust of the first and second welfare theorems, there are parallels in the literature. For example, in economies with increasing returns to scale, there are some allocations of a given total of initial endowments that are compatible with attainment of efficiency at a marginal cost-pricing equilibrium and some that are not (see Brown and Heal [1]). The orthogonality of efficiency and distribution might therefore be limited to "classical" economic environments free from increasing returns and public goods or externalities. In fact, there is a perspective from which increasing returns and public goods are closely related, so that this connection is not surprising.

3.5 Lindahl Permit Markets

In this section we compare the permits markets modeled previously in which there is a uniform price for all buyers and sellers, with a Lindahl-type framework in which each region may pay a different price for emission permits. This is motivated by reference to a Lindahl equilibrium, at which each producer of a public good is paid by every consumer for each unit produced, and in principle all consumers may pay different prices to a given producer. [25] In the present context the exact analog would be the following. Any region considering producing one more unit of emissions would have to purchase from every other the right to emit that extra unit. It would therefore have to buy an emission permit from each affected region, with possibly a different price ruling in each bilateral trade. This would give as many prices as there are in a Lindahl equilibrium.

[25] For a definition of Lindahl equilibria, see Foley [14] or Dasgupta and Heal [12].

An alternative way of interpreting such a model is to think of markets for externalities, as described by Meade [21] in his famous bees and apples example (see Dasgupta and Heal [12] for an exposition relevant to the present model). In this context each pairwise externality is a separate commodity, separately priced. There are therefore as many prices as there are pairs of interacting producers and consumers of externalities. In the present context, as the externalities imposed on a region depend only on the sum of emissions by other and not on the identities of the emitters, the dimensionality can be reduced so that the number of prices equals the number of regions rather than the number of pairs. There is a price for buying the right to pollute from each region that is the same for every buyer. At a normal Lindahl equilibrium, there are I^2 prices, one between each pair of the I regions, as each is both a buyer and a seller of emission rights, whereas with each charging a different price for a permit, there are only I prices. By comparison, in the framework modeled previously, there is only one price.

If each region faces a region-specific price for emission permits, the budget constraint (3.8) is changed to

$$\sum_l c_{i,l} p_l = \sum_l y_{i,l} p_l - p_{i,e}(E_i + a_i), \tag{3.13}$$

where $p_{i,e}$ is the price of an emission permit to i. Instead of (3.11), each region's first-order condition in production now becomes

$$\frac{\partial \Phi_i}{\partial y_{i,l}} = -\frac{p_l}{p_{i,e}}. \tag{3.14}$$

Recall that a necessary condition for efficiency is (3.6):

$$\overbrace{\frac{\partial \Phi_i}{\partial y_{i,l}} = \frac{-\dfrac{\partial u_i}{\partial c_{i,l}}}{\sum_k \lambda_k \dfrac{\partial u_k}{\partial a}} \; \forall l, \quad \text{and for } k \neq i, \quad \frac{\partial \Phi_k}{\partial y_{k,l}} = \frac{-\lambda_k \dfrac{\partial u_k}{\partial c_{k,l}}}{\sum_k \lambda_k \dfrac{\partial u_k}{\partial a}} \; \forall l,}^{\text{Lindahl-Bowen-Samuelson condition}}$$

so that in place of (3.12) the condition for permit markets to attain efficiency is

$$\frac{p_l}{p_{k,e}} = \frac{\lambda_k \dfrac{\partial u_k}{\partial c_{k,l}}}{\sum_k \lambda_k \dfrac{\partial u_k}{\partial a}} \; \forall k. \tag{3.15}$$

Because the permit price $p_{k,e}$ is region specific, this condition can now be satisfied without $\lambda_k(\partial u_k/\partial c_{k,l})$ being the same for all k. In other words this condition for Pareto efficiency can be satisfied now without an optimal distribution of income or wealth, which equates marginal valuations of consumption. Therefore, if redistribution of private goods or emission permits is ruled out, there is a real efficiency gain to having permit prices that are region specific, for without them it would not be possible to attain a Pareto-efficient allocation.

Appendix

Proof of Proposition 3

The first-order conditions for efficiency are

$$\overbrace{\frac{\partial u_i}{\partial c_{i,l}} = \lambda_k \frac{\partial u_k}{\partial c_{i,l}} \qquad \forall l = 1, ..., m \quad \text{and} \quad \forall k \neq i,}^{\text{equal marginal valuations of consumption}}$$

where region i is the designated region whose utility is being maximized, λ_k is a Lagrange multiplier associated with the constraint that region k reach a specified welfare level, and

$$\overbrace{\frac{\partial \Phi_i}{\partial y_{i,l}} = \frac{-\dfrac{\partial u_i}{\partial c_{i,l}}}{\Sigma_k \lambda_k \dfrac{\partial u_k}{\partial a}}\ \forall l, \quad \text{and for } k \neq i, \quad \frac{\partial \Phi_k}{\partial y_{k,l}} = \frac{-\lambda_k \dfrac{\partial u_k}{\partial c_{k,l}}}{\Sigma_k \lambda_k \dfrac{\partial u_k}{\partial a}}\ \forall l.}^{\text{Lindahl-Bowen-Samuelson condition}}$$

The first set of conditions, $\partial u_i/\partial c_{i,l} = \lambda_k(\partial u_k/\partial c_{k,l})\ \forall l, \forall k \neq i$, constitute a system of $(I - 1) \times m$ equations. If they are satisfied, then the second set of conditions is also satisfied at an equilibrium of a permit market. Therefore, we need to check only when the equal marginal valuation conditions are satisfied. Rewrite them as

$$\frac{\partial u_i}{\partial c_{i,l}} - \lambda_k \frac{\partial u_k}{\partial c_{k,l}} = 0. \tag{3.16}$$

Efficiency now requires that we locate a zero of a system of $(I - 1) \times m$ nonlinear equations given by (3.16).

What are the independent arguments of the functions in (3.16)? Note that once the prices of all goods are chosen, the production levels of private goods and of abatement are determined by equation (3.11), giving first-order conditions in production. And these levels, together with prices and endowments of permits, determine consumption levels through the budget constraint (3.8) and the first-order conditions on consumption (3.10). Therefore, the arguments of (3.16) can be taken to be E_i, $i = 1, ..., I$ and p_l, $l = 1, ..., m$ and e. Now, as the E_i are nonnegative and sum to a fixed number, they form a space of dimension $(I - 1)$. As there are only m relative prices, the left hand side of system (3.16) is a function, call it Ω, defined on $\Re^{(I-1)} \times \Re^{m} = \Re^{(I-1)+m}$. In fact it is defined on a subset of $\Re^{(I-1)+m}$ because if E is the vector of endowments and p the vector of relative prices, then $p = p(E)$: Equilibrium relative prices are determined by initial endowments. The graph of $p = p(E)$ is a subset of $\Re^{(I-1)+m}$ and indeed would be the equilibrium manifold of the economy under suitable regularity conditions.

The function Ω takes values in $\Re^{(I-1)\times m}$:

$$\Omega\colon \Re^{(I-1)+m} \to \Re^{(I-1)\times m},\ \Omega(x) = \frac{\partial u_i(x)}{\partial c_{i,l}} - \lambda_k \frac{\partial u_k(x)}{\partial c_{k,l}},$$

where $x \in \Re^{(I-1)+m}$. Proposition 3 uses the following regularity condition, which essentially states that the first-order conditions for efficiency in equation (3.5) change smoothly as prices and permit allocations change:

> *Regularity condition.* The matrix of first partial derivatives of the function Ω has full rank.

Note that Ω is defined on a compact set in $\Re^{(I-1)+m}$.

We now distinguish two cases: (1) $(I - 1) + m \leq (I - 1) \times m$ and (2) $(I - 1) + m > (I - 1) \times m$. In case 1 the dimension of the domain of Ω is less than or equal to that of the range, the regularity condition implies that the matrix of first partial derivatives is 1 to 1, and the compactness of the domain implies that the number of zeros of Ω is finite.

In case 2 the dimension of the domain exceeds that of the range. By basic transversality theory, the dimension of a preimage of zero is a manifold of codimension $(I - 1) + m - (I - 1) \times m > 0$ and is therefore a set of measure zero.

Note that an efficient equilibrium will be in the intersection of the graph of $p = p(E)$ with the zeros of Ω. In the case of two regions, there is a simple proof that this intersection is nonempty.

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